

Radio Design 201 Microwave Circuits and Antennas

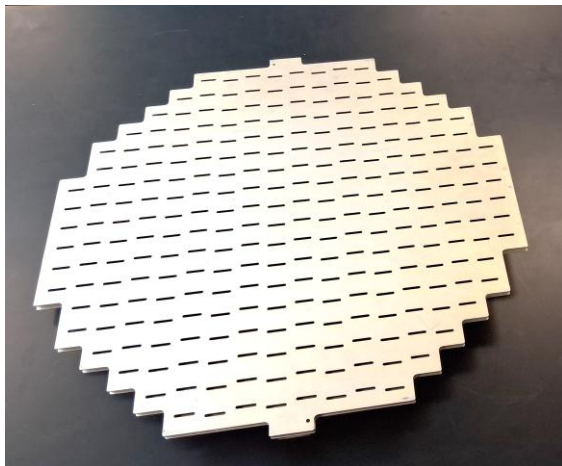
Episode 8 Phased-Arrays and Dish Antennas

Slides downloaded from: <https://ecefiles.org/>

Companion videos at: <https://www.youtube.com/playlist?list=PL9Ox3wpnB0kpOb7NdpHAK-jDpsZm3Xedx>

This material is **provided by ecefiles.org** for educational use .

This video provides a relatively comprehensive overview of how phased-array and parabolic dish antennas work. Derived from a university course, the material includes essential math formulas, but the presentation is intended for a wide audience from students, to radio amateurs, to practicing professionals. Several real-world examples are presented to provide a good grounding in this field.



Friis Equation(s)

Radio Design 201:

$$P_r = \frac{P_t G_t}{4 \pi d^2} A_r$$

(in free-space)

Classic Friis Equation:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4 \pi d} \right)^2$$

Large wavelength is "better" ??

Remember: $G = \frac{4 \pi A_e}{\lambda^2}$

Original Friis Equation:

$$P_r = P_t \frac{A_t A_r}{d^2 \lambda^2}$$

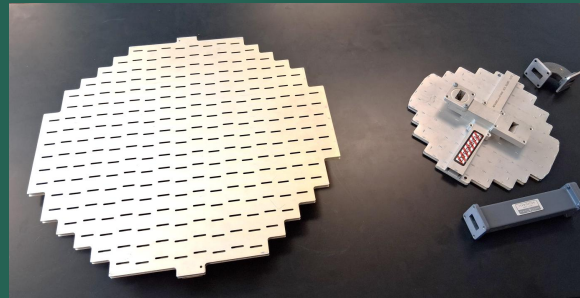
Small wavelength is "better"



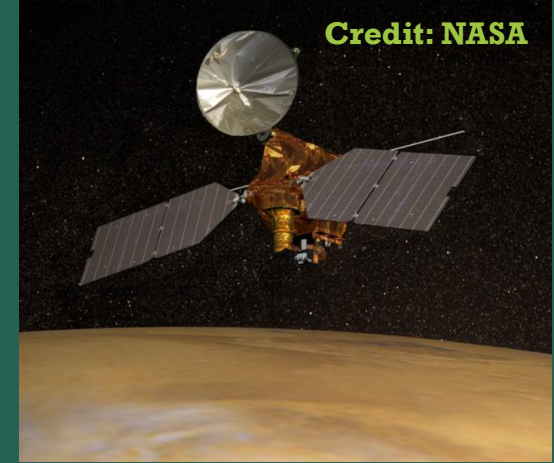
Credit: NASA

Microwave Circuits and Antennas

Radio Design 201 #8



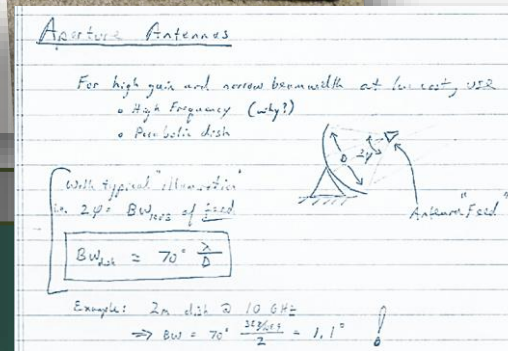
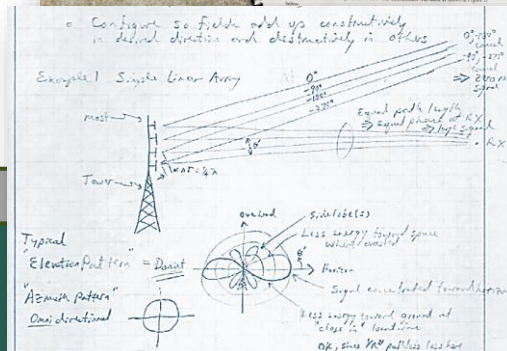
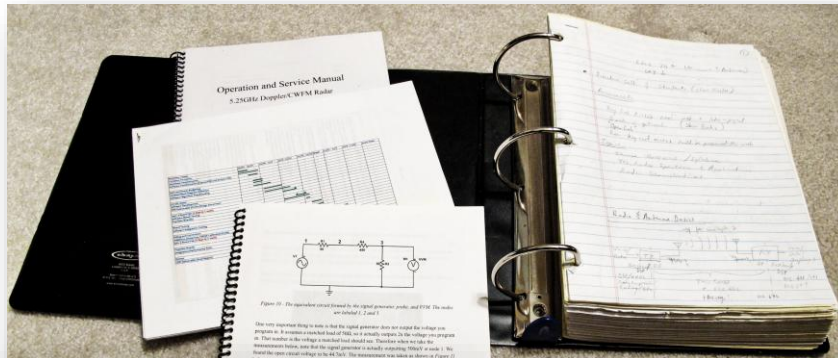
Phased-Arrays and Dish Antennas



Today's "Lecture" Topics

Announcements

Review material from prior Episodes



Topics

➡ **Phased Arrays (Theory and Measurement)**

Benefits of using Microwaves

Derivation of Beamwidth & Gain Formulas

- 1D Discrete Arrays
- Extension to 2D Aperture Antennas
- Dish antennas (Illumination-angle, gain, sidelobes, ...)
- Feeds (Horns, Helix, Dipole/corner-reflector, ...)

Application to Real-World Products/Design

Configure so fields add up constructively in desired direction and destructively in others

Example 1 Single Linear Array

most
 L
Tower

θ

$\theta = 0^\circ$
 -90°
 -180°
 -270°

Equal path length
 $\Rightarrow$ equal phase at RX
 $\Rightarrow$ high signal

RX

$\theta = 0^\circ$
 -90°
 -180°
 -270°

Side lobe(s)

Less energy toward space where overhead

Horizon

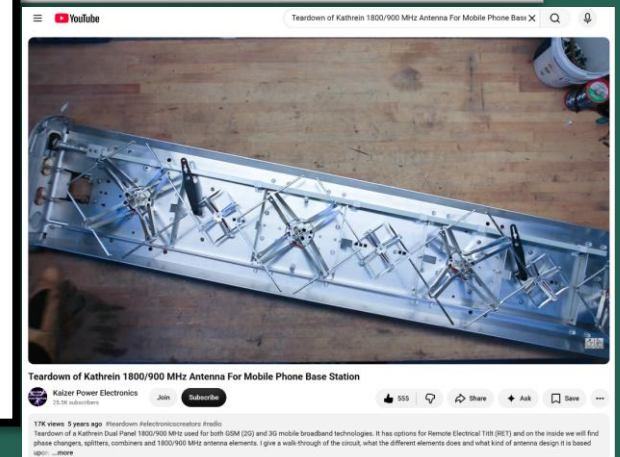
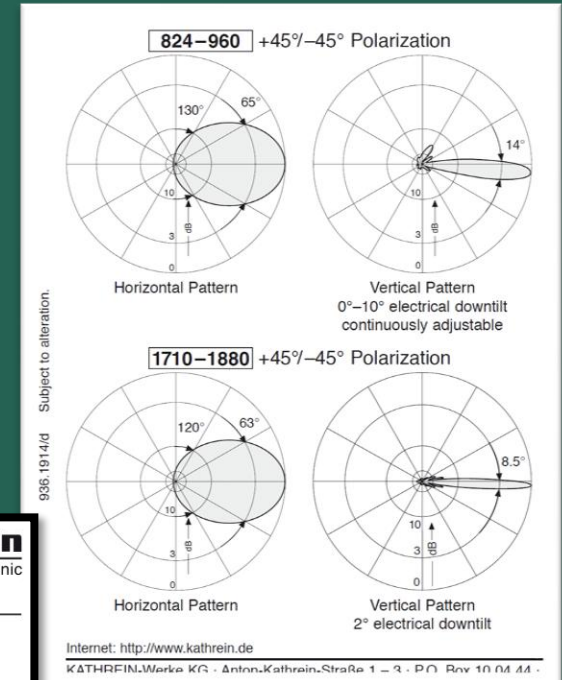
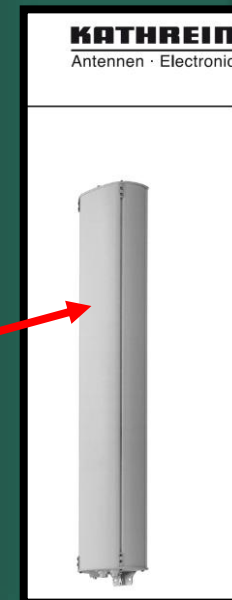
Signal concentrated toward horizon

Less energy toward ground at "close in" location

OK, since "VR" path loss less here

Typical "Elevation Pattern" = Donut

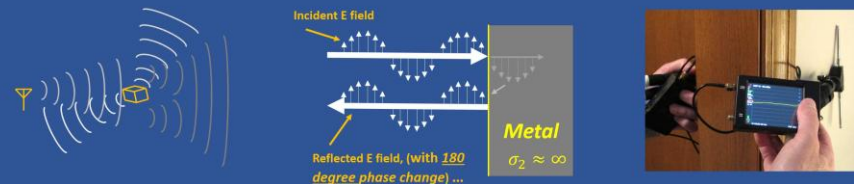
"Azimuth Pattern" = Omnidirectional

$$\theta_{3dB} \approx 50^\circ \frac{\lambda}{L}$$
$$\mathbf{G} \approx \frac{4}{\pi} (180^\circ)^2 \frac{1}{\theta_{el} \theta_{az}}$$


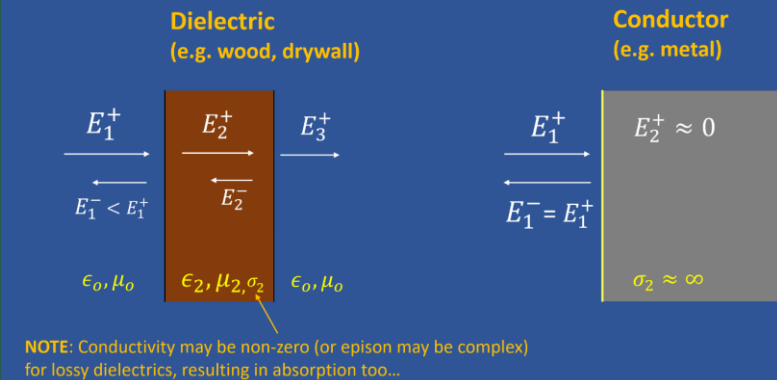
EM Wave Reflection

Antenna Briefs #7 – Part 2

Reflection of Radio Waves (and Diffraction, Absorption, ...)



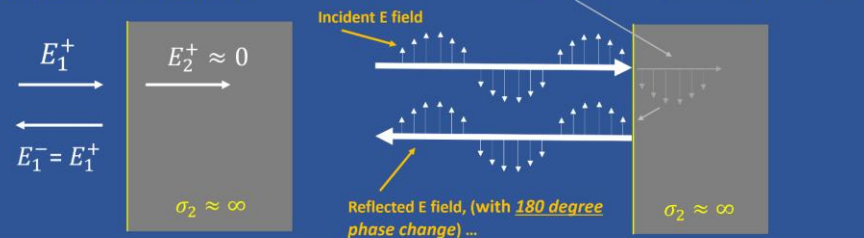
Undergrad EM Textbook View



How is Wave Reflected from Metal ?

$E_{\text{Tangential}} \rightarrow 0$ at surface of metal (else surface currents would be infinity)

RF currents induced in "skin" (surface) of metal, create reflected wave and stop transmission into the metal



Applications of Reflection Theory in Antenna Design



Microwave 1-D “Slot Array”



← → ↻ rigexpert.com/products/antennas/

Where to Buy

RigExpert Search

Gain	15 dBi
Polarization	Linear (Vertical)
Radiation pattern	H-Plane 90°, V-Plane 8°
Mounting	1/4" Screw
Dimensions	40mm x 25mm x 450mm

Where to Buy RigExpert MINI-GUN RigExpert reu-mini-gun.pdf Ask Gemini

dxengineering.com/parts/reu-mini-gun?srsltid=AfmBOort9ad7NRcUJ5TWw1fWCwJD4bwxRwVh4TcjzEbEKPZ...

DX ENGINEERING Keyword Search Deliver To: USA Change Location

RigExpert 5.8 GHz Minigun Directional UAV Antennas MINI-G... \$95.00 1 Add To Cart

★★★★★ (1) Part Number: REU-MINI-GUN Estimated Ship Date from Ohio: Today

Recommended Parts Overview Specifications Documentation Applications Reviews

Ease of Installation PriceValue Appearance Quality

5.0 5.0 5.0 5.0

Customer Images and Videos

Customer images and videos showing the antenna in use, including a graph on a screen and the antenna itself.

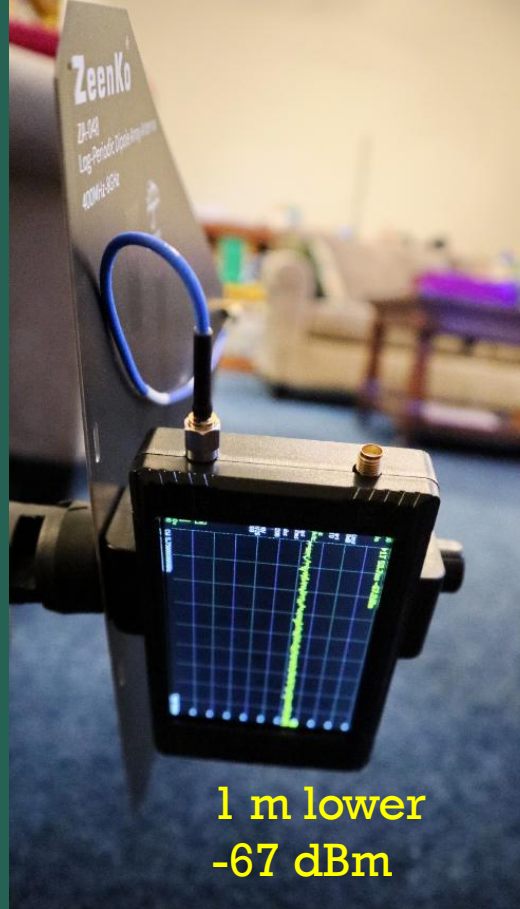
Home-Lab Indoor “Antenna Range” Measurements ☺



Transmitter is LiteVNA
in CW mode at 5.75
GHz



Receiver is TinySA Ultra in zero-span mode at
5.75 GHz with -50 dBm Ref level and 5 dB/div vert



Should be in the Far-Field, which
for this “electrically large”
antenna means a distance of:

$$D > \frac{2 L^2}{\lambda} = \frac{(2) 0.3m^2}{0.052m} = 3.5m$$

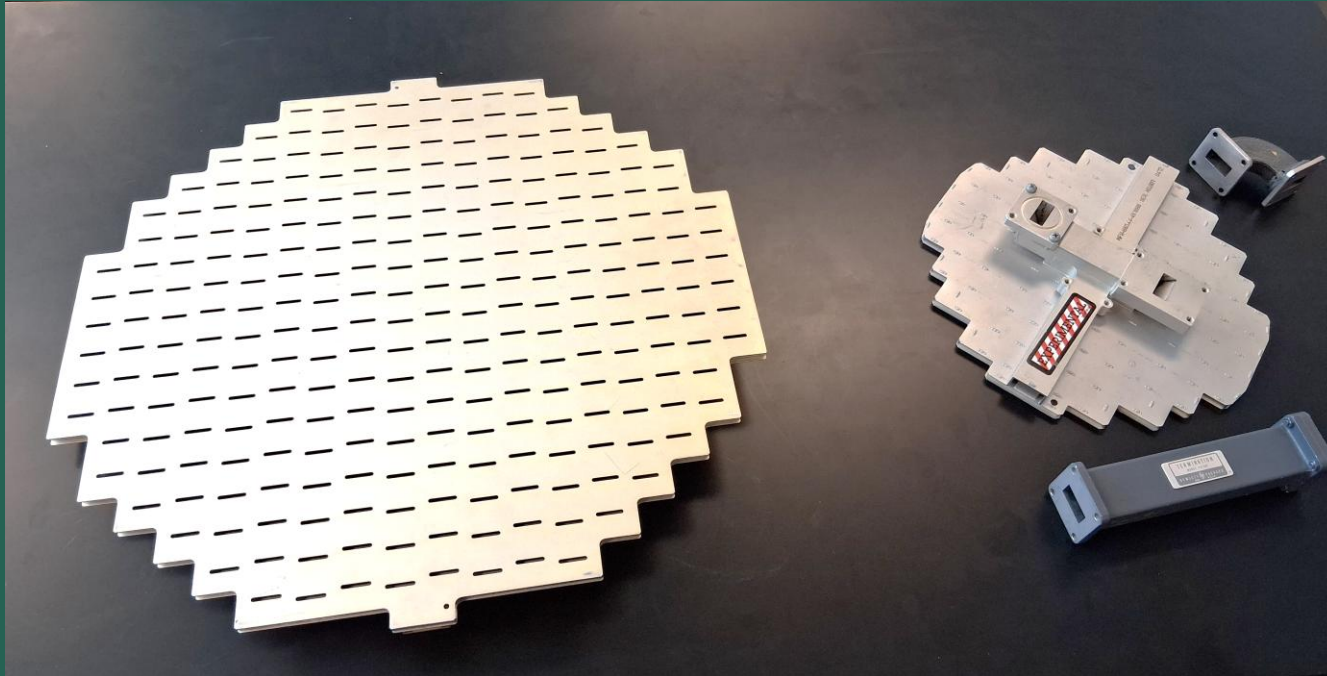
Need line of sight and minimal
multi-path reflections

We measured -14 dB at an elevation
angle of $\tan^{-1}(-1m/6m) = -13^\circ$

Receive antenna
is a wideband
log-periodic

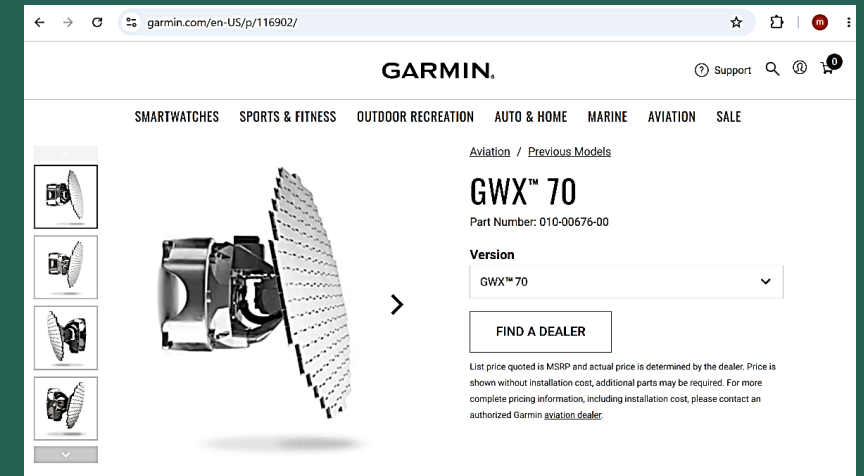


Microwave 2-D Slot Arrays



$$\theta \geq 50^\circ \frac{\lambda}{D}$$

$$G \approx (203^\circ)^2 \frac{1}{\theta_{el} \theta_{az}}$$



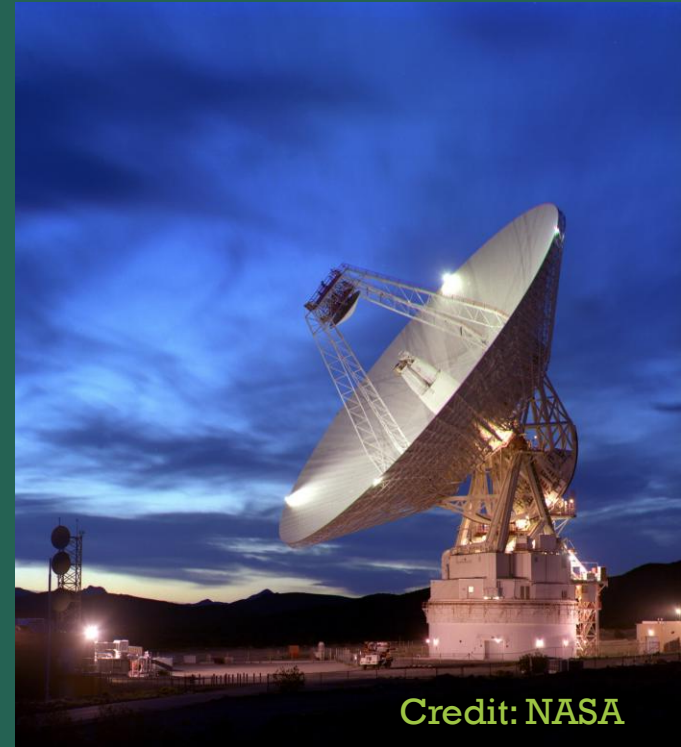
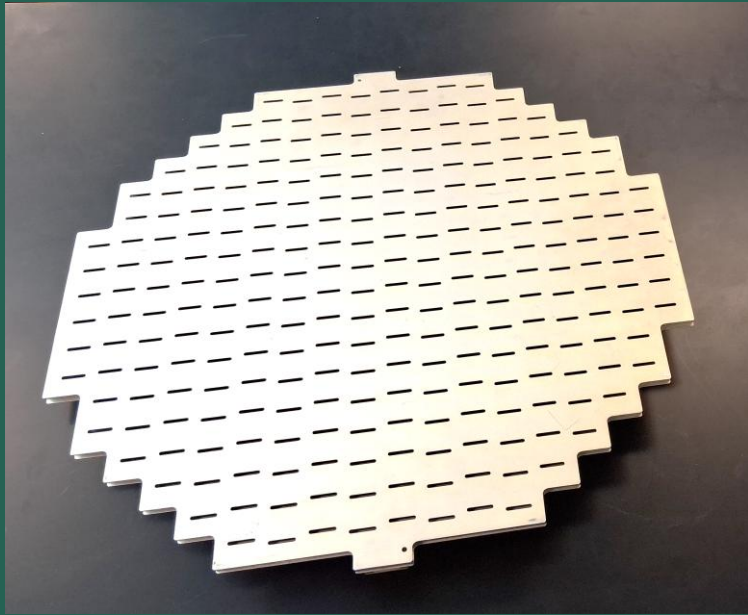
For Doppler Storm Tracking in Aircraft

Microwave Circuits and Antennas

Radio Design 201 #8

(Part 2)

Large- Aperture Antennas

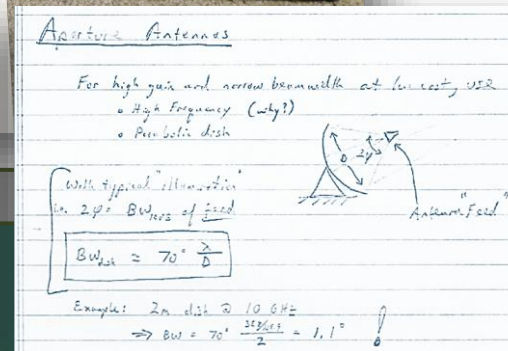
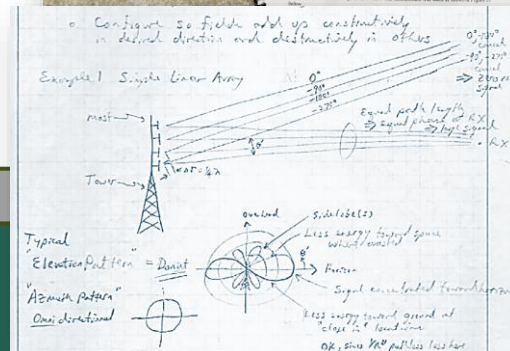
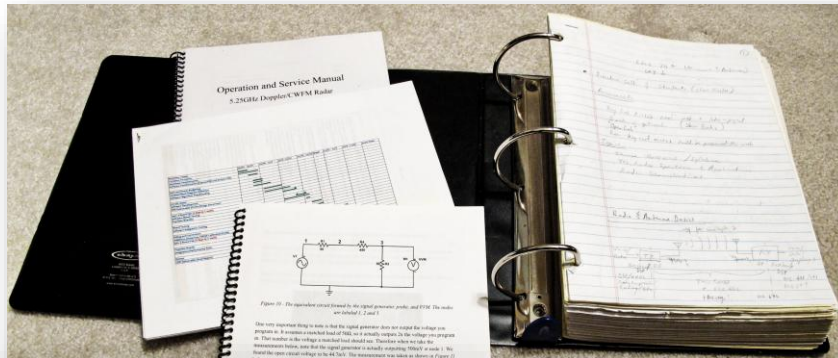


Credit: NASA

Today's "Lecture" Topics

Announcements

Review material from prior Episodes



Topics

Phased Arrays (Theory and Measurement)



Benefits of using Microwaves

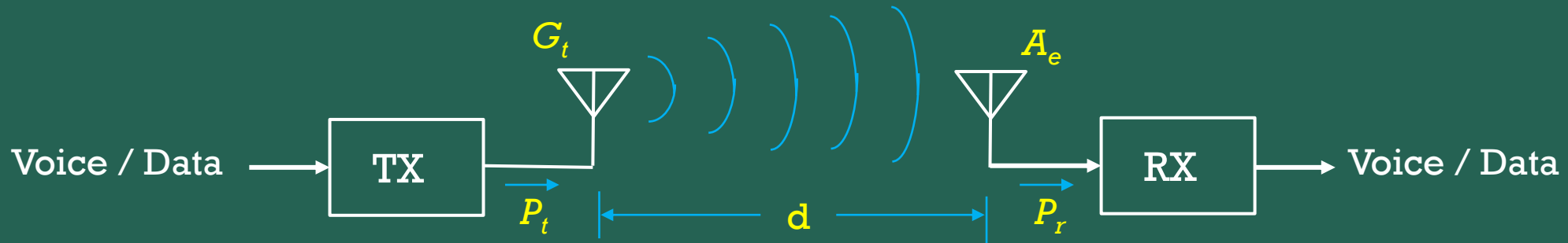
Derivation of Beamwidth & Gain Formulas

- *1D Discrete Arrays*
- *Extension to 2D Aperture Antennas*
- *Dish antennas (Illumination-angle, gain, sidelobes, ...)*
- *Feeds (Horns, Helix, Dipole/corner-reflector, ...)*

Application to Real-World Products/Design

Maximizing P_r

(comm-link analysis, from Episode 5)



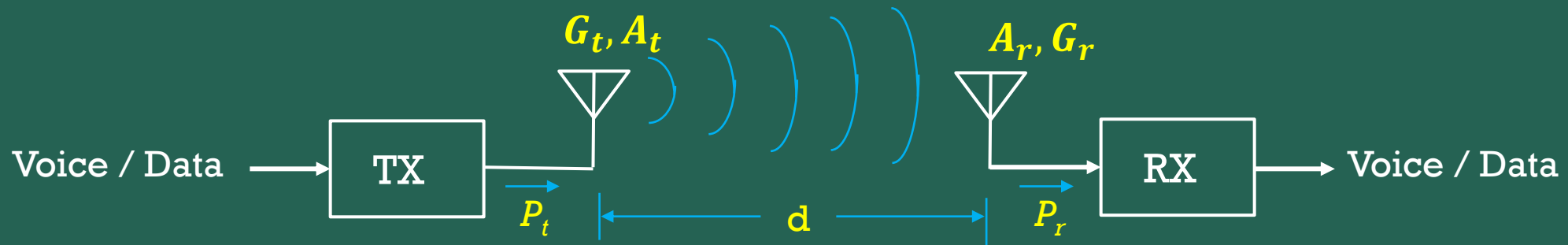
$$P_r = \frac{P_t G_t}{4 \pi d^n} A_e$$

Make these large for maximum range

Make this $n=2$

$\Rightarrow$ Use directional antennas,
microwave frequencies,
and line-of-sight

Friis Equation(s)



Radio Design 201:

$$P_r = P_t G_t \frac{1}{4 \pi d^2} A_r$$

(in free-space)

Classic Friis Equation:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4 \pi d} \right)^2$$

Large wavelength is “better” ??

Remember: $G = \frac{4 \pi A_e}{\lambda^2}$

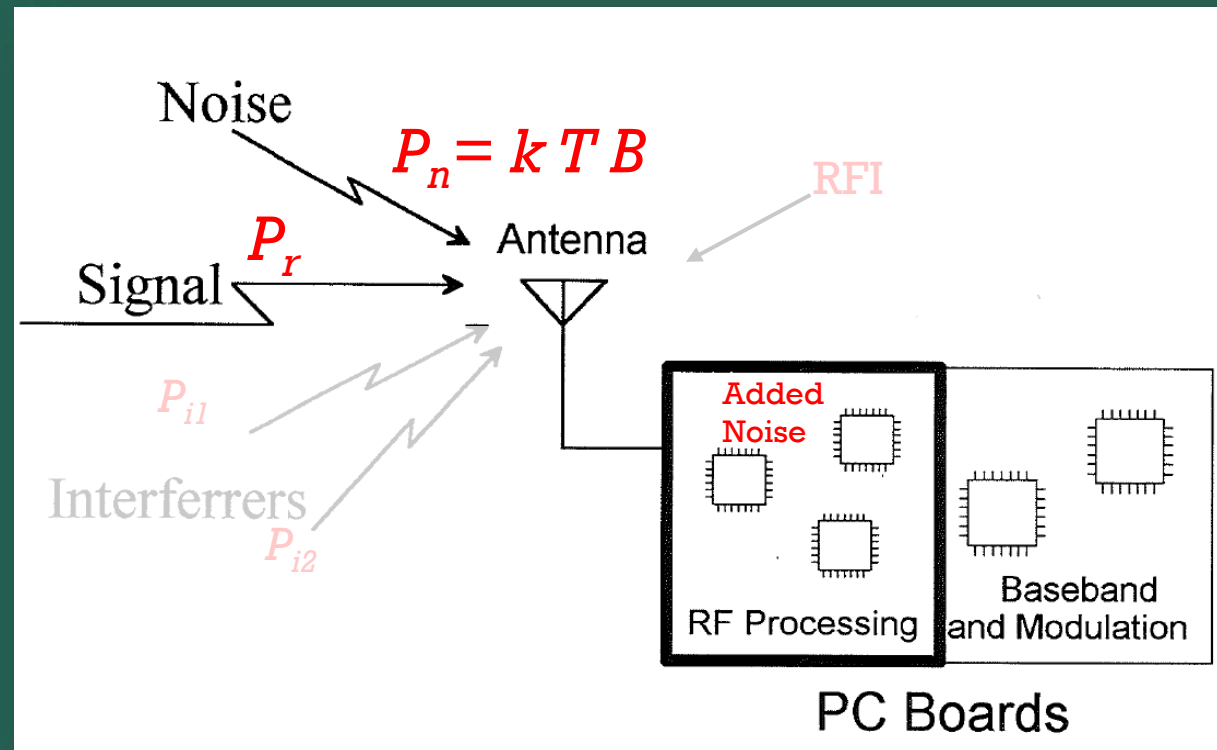
Original Friis Equation:

$$P_r = P_t \frac{A_t A_r}{d^2 \lambda^2}$$

Small wavelength is “better” ?

Noise Limits in Receivers

(from Radio Design 401, Episode 2)



Maximizing Range

(from Antenna Briefs, Episode 2)

Need:

$$P_r = P_t G_t \frac{1}{4 \pi d^n} A_e > (k T B)(SNR_{min})$$

- Use large transmit antenna gain G_t (if possible and legal*)
- Have a clear path when possible (line-of-sight, so “path loss exponent” $n = 2$)
- Use large receive effective aperture (large receive gain G_r)
- Decrease bandwidth B (and lower “antenna temperature” T) if possible
- Decrease required SNR (e.g. use digital modulation with source/channel coding)
- Increase transmit power P_t as last resort
- *Finally: Get licensed, and follow the rules ...

System-Level Example

$$P_r = P_t \frac{G_t}{4\pi d^2} A_e$$
$$> P_{noise} = kTB$$

$A_e \approx 2500 \text{ m}^2$
(@ 70% aperture efficiency)

Goldstone 70m DSN dish



Credit: NASA

230 Million Miles! ($d = 378E9$ meters)

$$P_r = 6.7E-15 \text{ W} \quad (-112 \text{ dBm})$$

$$kTB = 1.4E-16 \text{ W}^* \quad (-129 \text{ dBm})$$

$$SNR = 48 \quad (17 \text{ dB})$$

* Assuming system noise temperature $T = 21 \text{ K}$ and downlink bandwidth = 500 kHz (e.g. 500 kbps BPSK)



Credit: NASA

Mars Global Surveyor*

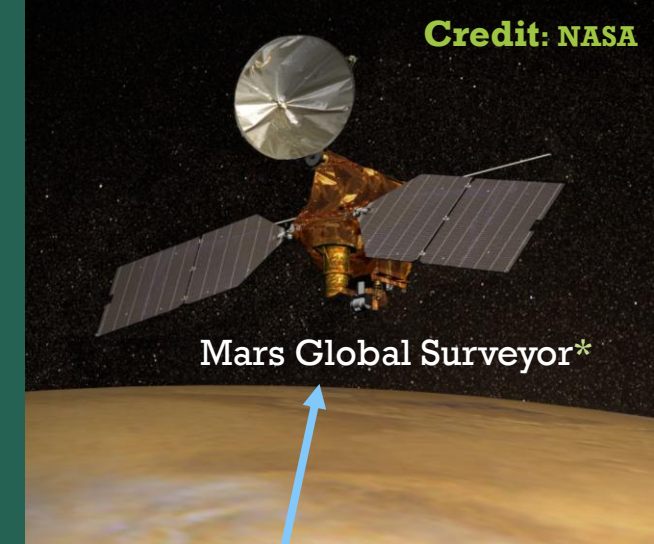
$P_t = 100 \text{ W}$
 $f = 8.4 \text{ GHz}$
 $\lambda = 0.036 \text{ m}$
3m dish
 $A_e = 4.9 \text{ m}^2$
 $G_t \approx 48,000$ (47 dB)
(@ 70% aperture efficiency)

*Rough estimates based on web search results

Rover Relay Operations

Rovers have a UHF whip antenna, and a Microwave dish.
UHF was used for most proximity data transmission.

Why?



While the monopoles are nominally linearly polarized with a toroidally shaped gain pattern, the structures on the spacecraft create significant distortions to both gain and polarization. This is especially true for the RUHF, due to the vertical obstructions (mainly the LGA and PMA) present on the deck. A right-hand polarization pattern, as measured on a rover mock-up in the JPL antenna range, is shown in Figure 3-10.

Figure 3-10 shows the RUHF antenna pattern in polar coordinates, with the concentric grid markers (0 to 120°) representing the cone angle (angle from the boresight) and the radial grid lines (0 to 360°) representing the clock angle. The RUHF pattern is not symmetrical with respect to the clock angle. The asymmetry causes significant variations in returned data volume from pass (orbiter overflight) to pass. The data-volume variations result mainly from

- The elevation profile of the orbiter and thus the pass duration,
- The azimuth profile of the orbiter during the overflight, and
- The rover orientation (tilt from horizontal) on the surface.

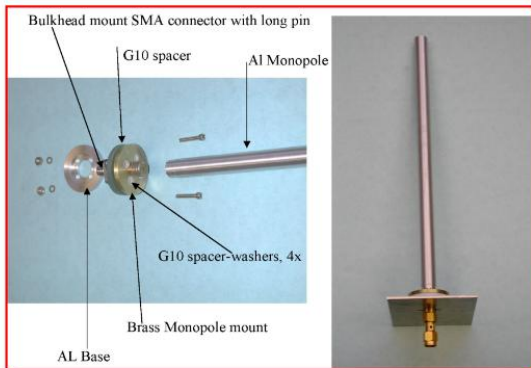


Figure 3-9. Rover UHF antenna.

29

Telecom Subsystem Hardware and Software

30

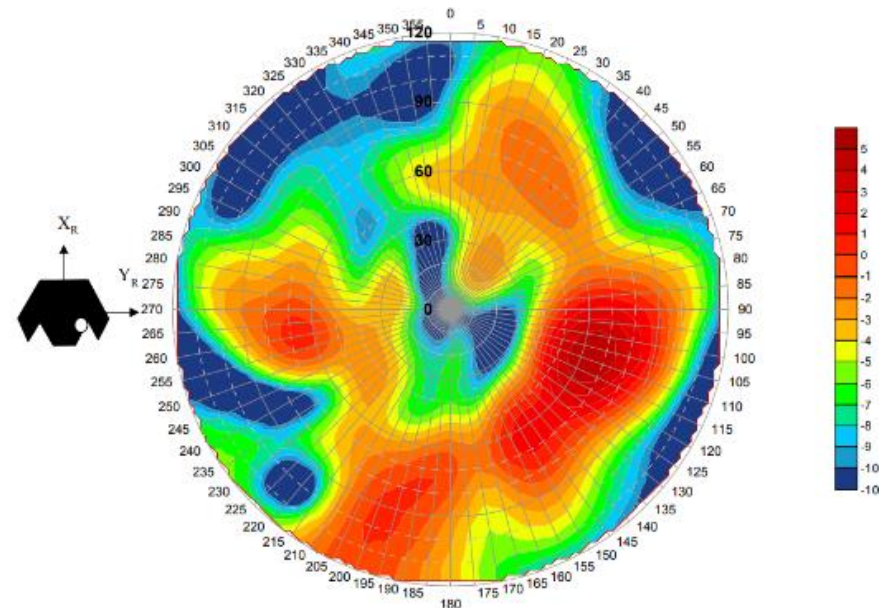


Figure 3-10. Rover UHF antenna pattern as measured on a mock-up at 402 MHz.



Interstellar Communication ?

Antenna Briefs #4A

Interstellar Comm and SETI (Revised)



Credit: NASA

ECEFILES.ORG



Credit: SETI.org

Homework #1

assigned in Episode 3 ☺



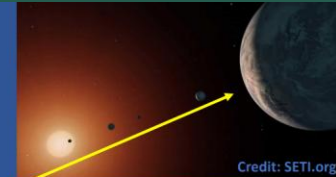
Credit: NASA



1. Could we communicate with ET in the TRAPPIST-1 star system ?
2. If so, with what bandwidth (and latency) ?

Assume 70m dishes at each end of link
Pt = 1 kW, 32 GHz, T = 20K, and BPSK w/ SNR_{min} = 10

ECEFILES.ORG



Credit: SETI.org

(Revised) Homework Solution

Need: $P_r = P_t \frac{G_t}{4\pi d^2} A_{eff} > (kTB) SNR_{min}$

Given:

Pt = 1000 W
Frequency 32 GHz
Earth-Trappist-1 distance: 39.5LY = 3.7E17 m
Transmit dish diameter: 70 m
Receiver dish diameter: 70 m
T = 20K, SNR_{min} = 10

Try: bandwidth = 1 Hz ...



Credit: NASA

Then:

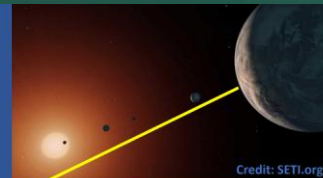
$P_r = 1.2E-21$ (-179 dBm)

$kTB = 2.8E-22$ W (-186 dBm)

$SNR = 4.3$ (+6 dB)

ECEFILES.ORG

229 trillion Miles



Credit: SETI.org

Some (approx.) intermediate values:

$$\lambda = \frac{c}{f} = 0.0094 \text{ m}$$

$$G_t = 5.5E8$$

$$A_{eff} = 3800 \text{ m}^2$$

Scaling to get the answer...

Recall SNR needs to be 10 or higher.

So we need to drop bandwidth by a factor of 10/4.3 = 2.3, to 0.4Hz

Latency is 2 x 39.5 years ☹

Some Technical References



Credit: NASA



Credit: JWST

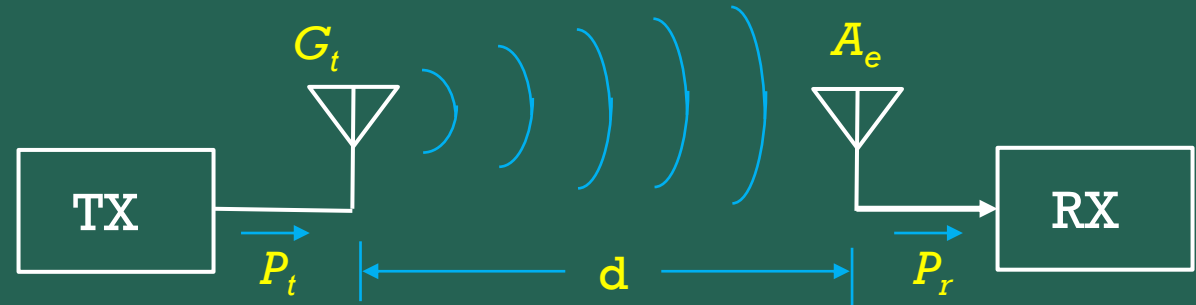
- Giuseppe Cocconi, and Philip Morrison, "Searching for Interstellar Communications", Nature, 1959.
- Bruce Murray, Samuel Gulkis and Robert E. Edelson, "Extraterrestrial intelligence - An observational approach", Science, 1978.
- Cordes and Woodruff, "Astrophysical coding: A new approach to SETI signals. I. Signal design and wave propagation", ASP Conf., 1995.
- Sheikh, et.al, "Choosing a Maximum Drift Rate in a SETI Search: Astrophysical Considerations", 2019. <https://arxiv.org/pdf/1910.01148.pdf>

ECEFILES.ORG

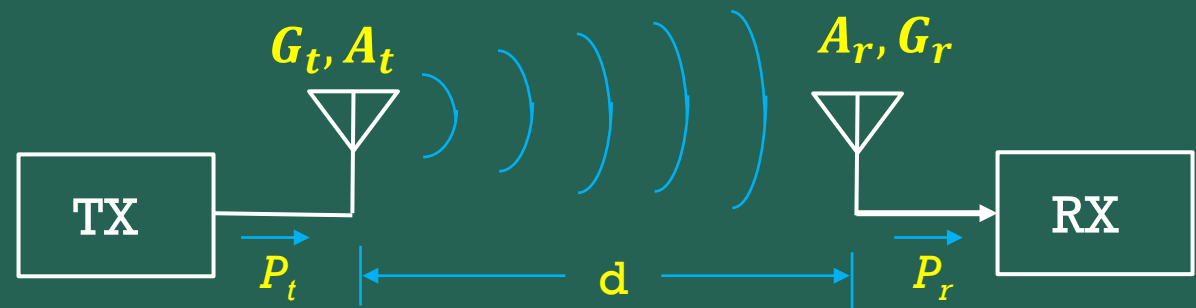
More on Gains and Effective Apertures

Some Definitions of Gain:

$$G_t \triangleq \frac{P_{\text{density-focused}}}{P_{\text{density-isotropic}}}$$



$$G_r \triangleq \frac{A_{e-RX\text{antenna}}}{A_{e-IsotropicAntenna}}$$



Transmit Antenna Gain Derivation



Transmit Antenna Gain

$$G_t \approx \frac{P_{\text{density-focused}}}{P_{\text{density-isotropic}}} = \frac{P_t / A_{\text{spot}}}{P_t / A_{\text{sphere}}} = \frac{A_{\text{sphere}}}{A_{\text{spot}}} = \frac{4\pi R^2}{(R \theta_{az})(R \theta_{el})} = \frac{4\pi}{\theta_{az} \theta_{el}}$$

Approximate since angles are typically 3dB, not sharp cutoffs

If azimuth and elevation angles are in degrees:

$$G_t \approx \frac{4\pi}{\theta_{az} \frac{\pi}{180^\circ} \theta_{el} \frac{\pi}{180^\circ}} = \frac{4}{\pi} (180^\circ)^2 \frac{1}{\theta_{az} \theta_{el}} = \frac{41,253}{\theta_{az} \theta_{el}}$$

Reciprocity Theorems



[AI Mode](#) [All](#) [Videos](#) [Images](#) [Shopping](#) [Forums](#) [Short videos](#) [More](#) [Tools](#)

◆ AI Overview

The [Reciprocity Theorem in electromagnetism](#) states that the transmission and reception characteristics of an antenna are identical if the medium between them is linear, passive, and isotropic.  YouTube · Padmasri Na... +1

Key Consequences for Antennas

- **Equality of Radiation Patterns:** An antenna has the exact same directional pattern and directivity whether it is transmitting or receiving.  YouTube · Engineering F... +1
- **Equality of Impedances:** The terminal impedance of an antenna is identical in both transmitting and receiving modes.  YouTube · Engineering F... +1
- **Equality of Effective Length/Height:** An antenna maintains the same effective length or effective aperture for transmission and reception.  YouTube · Engineering F... +1
- **Symmetric Power Transfer:** The ratio of received power to transmitted power between two antennas remains unchanged if their roles are completely interchanged.  Cadence +1

Conditions for Validity

The theorem relies on specific assumptions about the system and environment:

- **Linearity:** The media and materials must respond linearly to field strengths.
- **Passivity:** No active devices (like internal amplifiers) can be part of the reciprocal path.
- **Isotropy:** The properties of the propagation medium must be uniform in all



 YouTube · Padmasri Naban

Reciprocity Theorem for Antennas with Its Proof

Dec 14, 2020 — The video discusses the reciprocity...



 YouTube · Engineering Funda

Reciprocity Theorem (Definition, Basics,...

Apr 26, 2018 — Reciprocity Theorem (Definition, Basics,...



 Scribd

Reciprocity Theorem in Antenna Theory | PDF - Scribd

The Reciprocity Theorem states that an antenna behaves identically for both...



[Show all](#)

Important Take-Homes



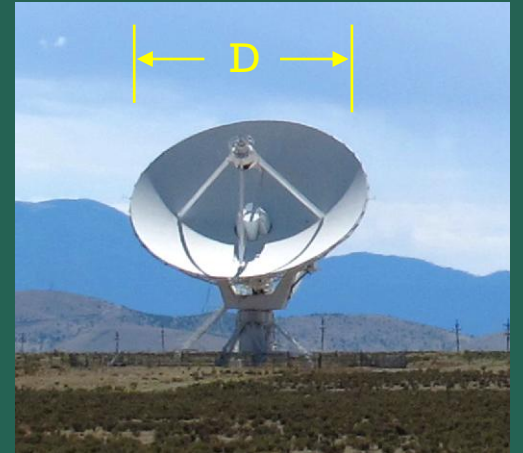
Simple half-wave dipoles are relatively isotropic, but have small receiving apertures at microwave frequencies:

$$G \approx 1.6 \text{ (2 dBi)}, \quad \text{and} \quad A_e \approx G \frac{1}{\pi} \left(\frac{\lambda}{2} \right)^2$$

Aperture antennas can have very large gain at microwave frequencies:

$$A_e \approx \eta \pi \left(\frac{D}{2} \right)^2 \quad \text{and} \quad G = \frac{A_e}{A_{ei}} = \frac{4\pi A_e}{\lambda^2}$$

Aperture Efficiency (e.g. 0.7)



But must be carefully pointed, since $\theta_{3dB} \approx 70^\circ \frac{\lambda}{D}$

Microwave Circuits and Antennas

Radio Design 201 #8

(Part 3)

$$\left[\sum_{n=0}^{N-1} I_n e^{-j n \Omega} \right]$$

Array Factor

Directional Antennas



Theory and Practice

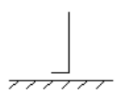
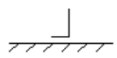
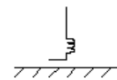
Basic Non-directional* Antennas

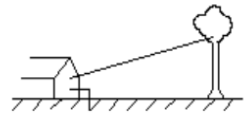
See Episode 7: “Antennas and Feedline Systems”

Antennas come in a nearly infinite number of shapes and sizes. Here are a few of the more common ones. Note that many commercial designs are variations or combinations of these.



“Non-directional” Types

Antenna	Impedance	Features
1) Halfwave Dipole	 73+j0 Ohms	Relatively isotropic, simple construction
2) Folded (halfwave) Dipole	 $300 + j0$ Ohms	Similar to dipole. Higher impedance. Sometimes used in simple vertical arrays.
3) Quarterwave Monopole	 $36+j0$ Ohms	Similar to dipole. Ground-plane often abbreviated (e.g. case of cell-phone is ground plane)
4) Short Monopole	 $R - jX$ $R \ll 36, X \text{ large}$	Physically shorter than quarterwave monopole, but requires resonating coil and/or matching network.
5) Loaded monopole	 $R + j0$	Similar to short monopole, but resistive input impedance. Can be engineered to be nominally 50 Ohms.

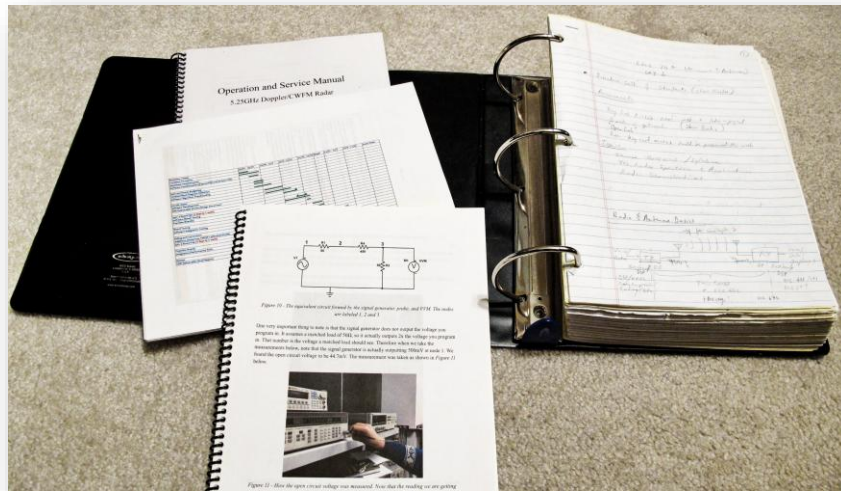
6) Simple longwire		Varies Widely	Simple useful design at low freq (e.g. < 30 MHz). Often used for shortwave receivers.
7) Simple large loop or smaller resonant loop		Varies Widely	Simple, low-cost. Popular for indoor UHF TV antennas in "the old days".. Resonant loop used in KeyFobs today.
8) Small ferrite-core loop		$R + jX$	Physically small with large effective aperture. Used in portable AM broadcast receivers and other LF to HF products.
9) Microstrip patch		50 Ohms	Simple, low-cost. Useful mainly at high frequency (e.g. good for GPS).
10) Other	Inverted F, small loaded patches, etc.	50 Ohms	Simple, low-cost designs for PC boards. May use high-K dielectrics to make antenna much smaller than a wavelength.

* NOTE: All antennas have some directivity

Today's “Lecture” Topics

Announcements

Review material from prior Episodes



Topics

✓ **Phased Arrays** (Theory and Measurement)

✓ **Benefits of using Microwaves**

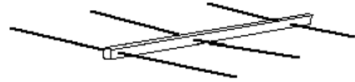
➡ **Derivation of Patterns and Beamwidths**

- **1D Discrete Arrays and Fourier Transform Effects**
- **Extension to 2D Aperture Antennas**
- **Dish antennas (Illumination-angle, gain, sidelobes, ...)**
- **Feeds (Horns, Helix, Dipole/corner-reflector, ...)**

Application to Real-World Products/Design

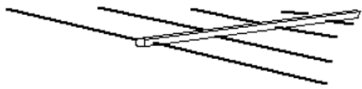
Directional Antennas and Basic Linear Array

1) Yagi-Uda



Moderate gain (10 dB)
Good front-to-back ratio.
Relatively simple construction.

2) Log-Periodic



Similar to Yagi-Uda, but
broadband. Lower gain
and less directivity.

3) Corner Reflector



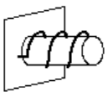
Good "sector-coverage" (i.e.
beamwidth of 90 to 120 degrees
with excellent front/back ratio).
Often used in cell-towers.

4) Horn



Gains to about 12 dB. Good
illumination pattern for dish
antennas.

5) Helix



Circular polarization. Moderate
gain, and good illumination for
dish antenna.

6) Linear, vertical array
(tower-mounted dipoles)



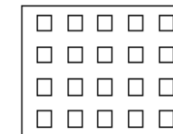
Concentrates power toward
horizon for max range. Used
in public-safety (police/fire/etc.)
Used in cell-towers when array
embedded in corner reflector.

7) Parabolic dish



High-gain, narrow-beamwidth.
Simple, low-cost construction.

8) 2-D Phased-array
Array of microstrip patches,
or waveguide slots



High-gain, narrow-beamwidth,
rapid-steering. Relatively high
cost.

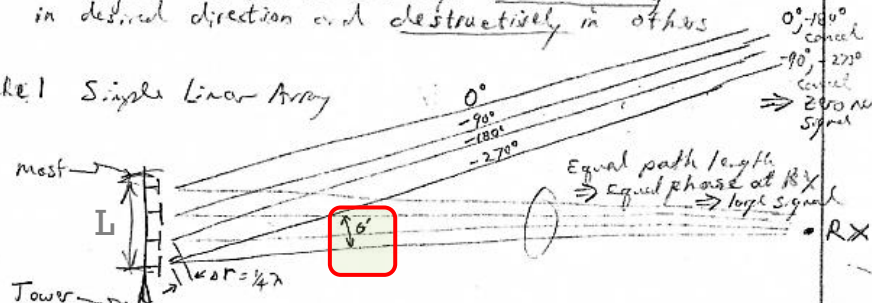
1-D Array Analysis (the math)

Qualitative Description

Basic Concept:

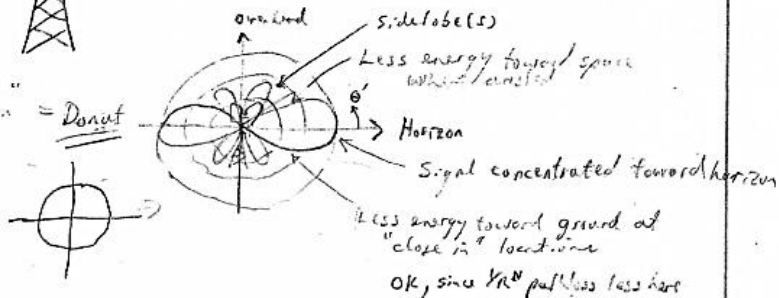
- Use multiple (or distributed) sources
- Configure so fields add up constructively in desired direction and destructively in others

Example 1 Simple Linear Array



Typical
"Elevation Pattern"
= Donut

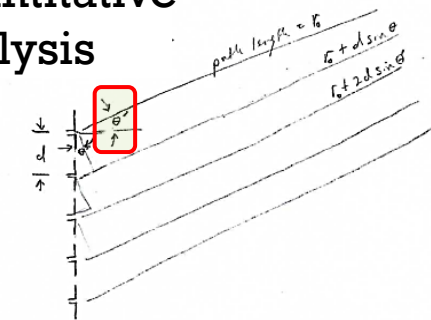
"Azimuth Pattern"
Omnidirectional



Quantitative Analysis

Element #

0
1
2
3
4



Field at observation point (r, θ) from element n is (for short dipole)

$$E_n(r, \theta) \approx 30 I_n L \beta \cos \theta' \frac{1}{r_n} e^{-j\beta r_n} \quad \text{where } \beta = \frac{2\pi}{\lambda} \quad r_n = r_0 + nd \sin \theta$$

$$\text{OR, In general, } = F(\theta) \frac{1}{r_n} I_n e^{-j\beta r_n}$$

And total field is

$$\begin{aligned} E_{\text{tot}} &= \sum_{n=0}^{N-1} F(\theta) \frac{1}{r_n} I_n e^{-j\beta r_n} \\ &= F(\theta) \sum_{n=0}^{N-1} \frac{1}{r_0 + nd \sin \theta} I_n e^{-j\beta(r_0 + nd \sin \theta)} \\ &\approx \left(F(\theta) \frac{1}{r_0} e^{-j\beta r_0} \right) \sum_{n=0}^{N-1} I_n e^{-j\beta nd \sin \theta} \end{aligned}$$

Field of single element "Array Factor"



NOTES: Array factor $\sim$ (discrete) Fourier Transform of excitation currents I_n $\psi' \text{ p.d.s.}(\theta) = \Omega$

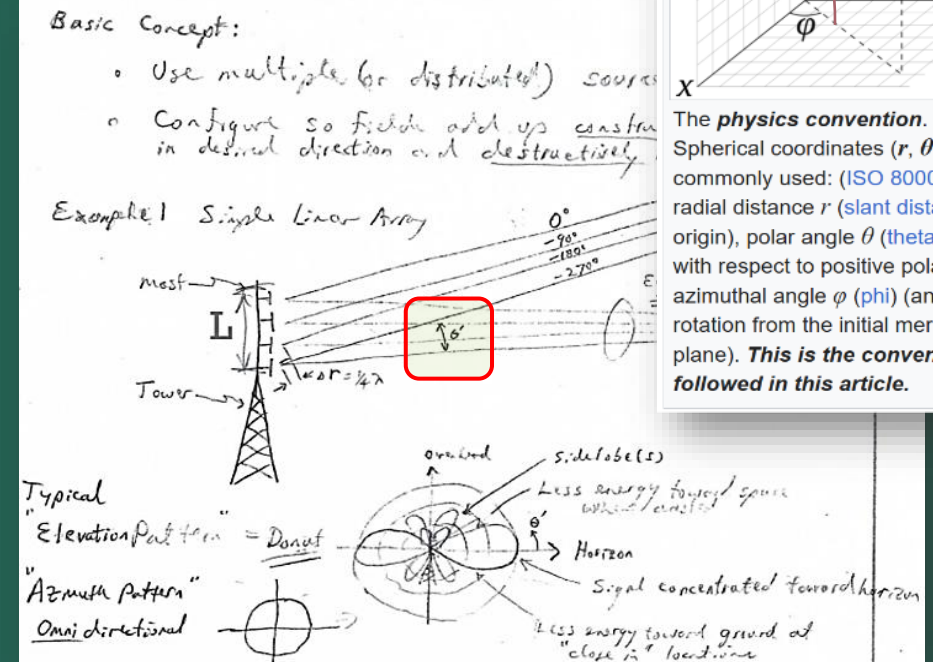
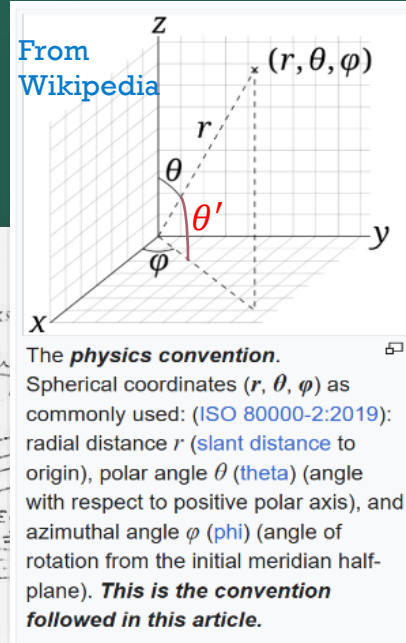
$\rightarrow$ Can pick I_n to give desired pattern using FT theory!

Fourier Transform Implications

$$E(\theta', \phi) \approx \left[\underset{\substack{\uparrow \\ \text{Element} \\ \text{Factor}}}{F(\theta', \phi)} \frac{1}{r_o} \overbrace{e^{-j 2\pi \frac{r_o}{\lambda}}}^{1 \angle \left(-2\pi \frac{r_o}{\lambda}\right)} \right] \left[\sum_{n=0}^{N-1} I_n e^{-j n \Omega} \right] \quad \text{where} \quad \Omega = 2\pi \frac{d}{\lambda} \sin(\theta') \approx 2\pi \frac{d}{\lambda} \theta'$$

Propagation Factor
Array Factor = Discrete Time Fourier Transform (D T F T) of I_n values

- More elements (larger N , longer sequence)
=> Narrower beamwidth
- Tapering off I_n amplitudes at ends of array
=> Lower sidelobes
- Varying I_n phases linearly ($\angle I_n = n \Delta\phi$) shifts pointing direction
=> Beam steering !



Endfire vs Broadside Arrays

How can the far field pattern of an array antenna system be found using the discrete time Fourier transform of the individual antenna excitation phasors?

Core Idea

The far-field pattern of an array is found by treating the set of complex excitation phasors I_n (amplitude and phase driving each element) as a discrete sequence in space, and computing its discrete-time Fourier transform (DTFT) evaluated along a spatial-frequency variable tied to the observation angle. This works because superposition of radiated fields from equally-spaced, identical elements produces exactly the same summation structure as the DTFT of a sampled signal. [onlinelibrary.wiley +1](#)

Setting Up the Array Factor as a DTFT

For a linear array of N identical elements spaced by d along an axis, each driven by phasor I_n , the total far field is the vector sum of the individual element fields, each phase-shifted by its position:

$$E(\theta, \phi) = E_{el}(\theta, \phi) \sum_{n=0}^{N-1} I_n e^{jk nd \cos \phi}$$

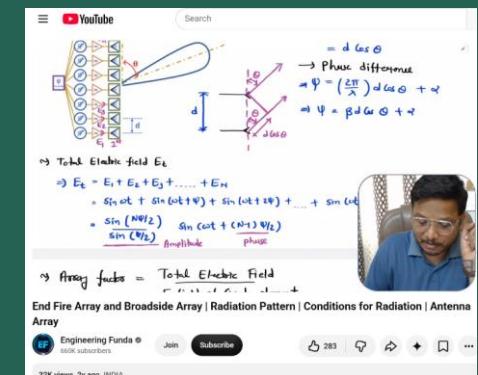
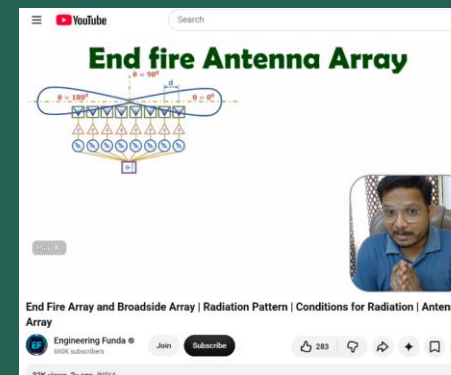
where E_{el} is the single-element pattern and $k = 2\pi/\lambda$ is the free-space wavenumber. Define the spatial frequency variable [engineering.purdue +1](#)

Good. This matches what I have seen, but the use of cosine phi is awkward and does not allow the second equation to be simplified using a small angle approximation. What is the geometry of the array here, relative to the angle phi?

Summary of the Two Conventions

Convention	Angle reference	Phase term	Small-angle beam approx. valid at
Endfire-style (my earlier notes)	ϕ from array axis	$kd \cos \phi$	ϕ near 0° or 180° (endfire)
Broadside-style	θ from broadside normal	$kd \sin \theta$	θ near 0° (broadside)

Both describe the identical physical array — the difference is purely which reference axis you measure the angle from. If your array's main beam is broadside (as most amateur-radio linear arrays and typical phased-array apertures are), switching to the θ -from-broadside convention is the right move, since it puts $\sin \theta$ directly in the exponent and lets you Taylor-expand around $\theta = 0$ for beamwidth, sidelobe-location, or scan-loss approximations. [ecen665web.groups.et.byu](#)



Ask a follow-up



Search



Computer

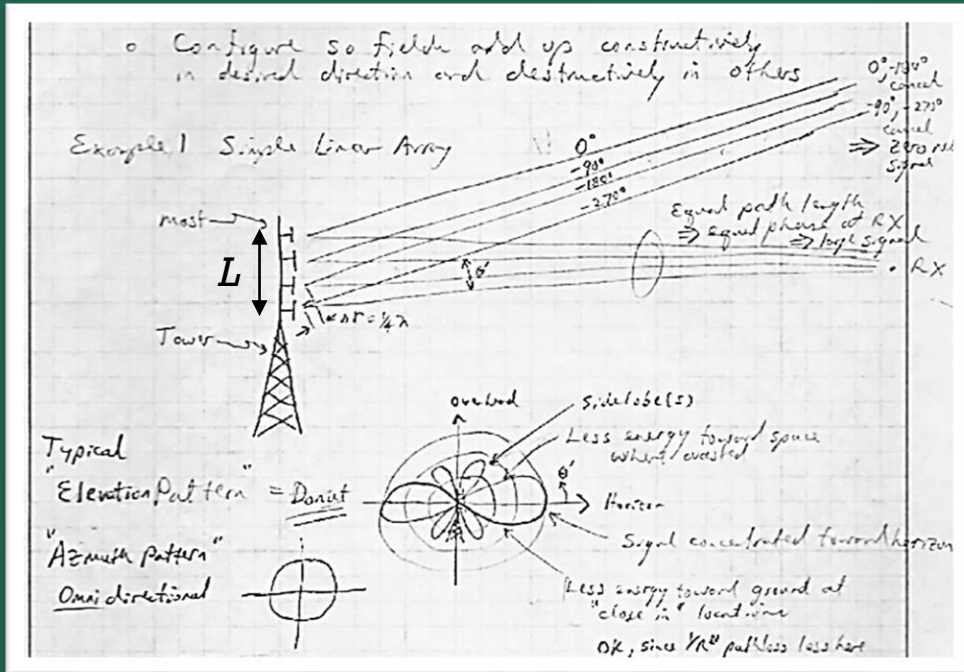
Sonnet 5 Thinking



<https://www.youtube.com/watch?v=EtlcXlYaCbc>

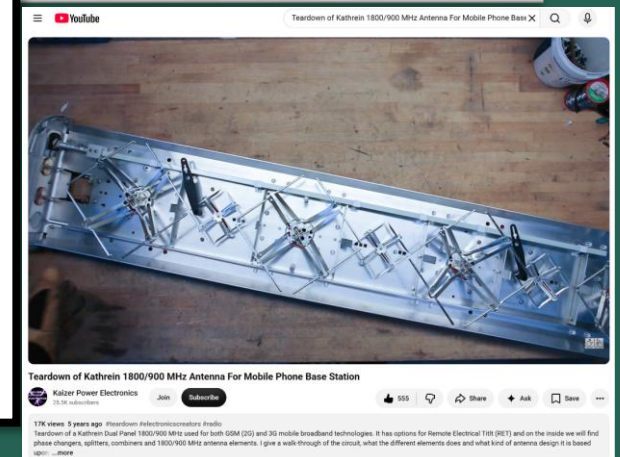
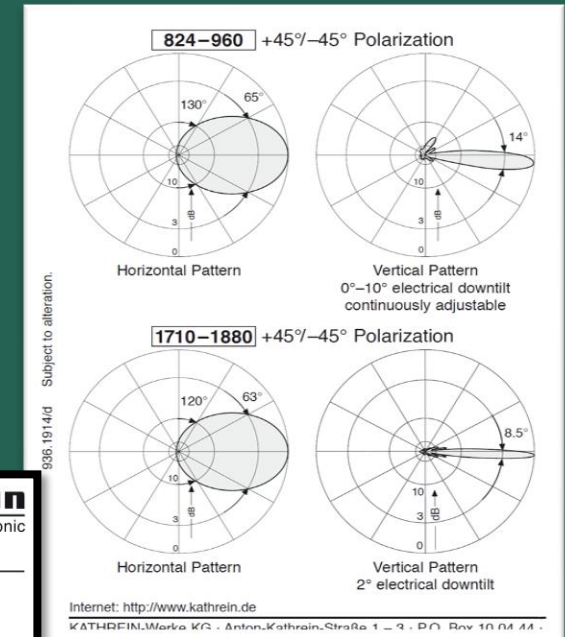
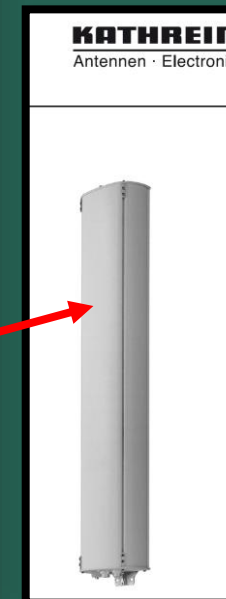
Pattern and Beamwidth

(See Part 2 of this Episode)



Beamwidth

$$BW_{3dB} \approx 50^\circ \frac{\lambda}{L}$$



Beamwidth Formula Derivation

Examine Array Factor (substitute $\frac{2\pi}{\lambda} = \theta$)

$$\sum_{n=0}^{N-1} I_n e^{-j n 2\pi \frac{d}{\lambda} \sin \theta'}$$

with $I_n = I_0 \forall n$, $\frac{d}{\lambda} = 0.5$. (Uniform excitation of elements and minimum spacing)

$$\Rightarrow AF = I_0 \sum_{n=0}^{N-1} e^{-j n \pi \sin \theta'}$$

To simplify sum, let $\sin \theta' = r$

$$\Rightarrow AF = I_0 \sum_{n=0}^{N-1} e^{-j n \pi r}$$

$$= I_0 \sum_{n=0}^{N-1} e^{-j n \pi r} = \sum_{n=0}^{N-1} e^{-j n \pi r}$$

$$= I_0 \sum_{n=0}^{N-1} e^{-j n \pi r} [1 - e^{-j N \pi r}]$$

$$= I_0 \frac{1 - e^{-j N \pi r}}{1 - e^{-j \pi r}}$$

$$= I_0 \frac{e^{-j \frac{N}{2} \pi r} (e^{j \frac{N}{2} \pi r} - e^{-j \frac{N}{2} \pi r})}{e^{-j \frac{\pi}{2} r} (e^{j \frac{\pi}{2} r} - e^{-j \frac{\pi}{2} r})}$$

$$= I_0 e^{-j \frac{N-1}{2} \pi r} \frac{\sin(\frac{N}{2} \pi r)}{\sin(\frac{\pi}{2} r)}$$

$$= I_0 e^{-j \frac{N-1}{2} \pi r} \frac{\sin(\frac{N}{2} \pi r)}{\sin(\frac{\pi}{2} r)}$$

$$= I_0 e^{-j \frac{N-1}{2} \pi r} \frac{\sin(\frac{N}{2} \pi r)}{\sin(\frac{\pi}{2} r)}$$

$$= I_0 e^{-j \frac{N-1}{2} \pi r} \frac{\sin(\frac{N}{2} \pi r)}{\sin(\frac{\pi}{2} r)}$$

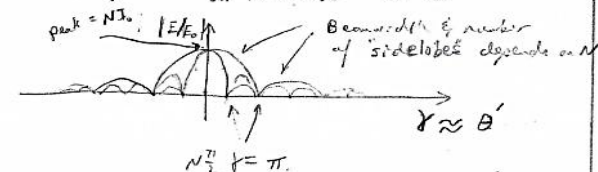
$$= I_0 e^{-j \frac{N-1}{2} \pi r} \frac{\sin(\frac{N}{2} \pi r)}{\sin(\frac{\pi}{2} r)}$$

$$= I_0 e^{-j \frac{N-1}{2} \pi r} \frac{\sin(\frac{N}{2} \pi r)}{\sin(\frac{\pi}{2} r)}$$

$$= I_0 e^{-j \frac{N-1}{2} \pi r} \frac{\sin(\frac{N}{2} \pi r)}{\sin(\frac{\pi}{2} r)}$$

Recall $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$

$\Rightarrow$ Pattern = a sinc(.) function



$$\gamma_{1st null} \approx \theta' = \frac{2}{N} \text{ radians} \Rightarrow \frac{110^\circ}{N}$$

Example Suppose $N = 8$

$\Rightarrow$ First null when

$$\theta' \approx \gamma = \frac{2}{N} = \frac{2}{8}$$

$$\Rightarrow = 0.25 \text{ radians}$$

$$= 15^\circ$$

$\Rightarrow$ Null-Null Beamwidth $\approx 30^\circ$ (actually 29°)

3dB point reached when

$$\sin(N \frac{\pi}{2} \gamma) \approx 0.707 N$$

$$\Rightarrow \gamma \approx 0.11$$

value of AF at $\gamma = 0$

(to give $N \frac{\pi}{2} \gamma = \pi/4$)

$$\theta = \sin^{-1}(0.11) = 0.1102 \approx 0.11$$

$$= 6.3^\circ$$

$\Rightarrow$ 3dB Beamwidth $\approx 12.5^\circ$ for $N=8$

(Compare $\omega \approx 90^\circ$ for single dipole element)

Note

Doubling # of elements halves beamwidth
Halving # of elements doubles beamwidth

N	BW _{3dB}	Array Dimension $L = N \frac{\lambda}{2}$
8	12.5°	4λ
16	6.3°	8λ
4	25°	2λ
2	50°	1λ

In general, (For uniform excitation & $N > 2$)

$$BW_{3dB} \approx 50^\circ \frac{\lambda}{L}$$

Total array dimension
 $= N \frac{\lambda}{2}$ for min spacing

2-D Array or Dish Beamwidth

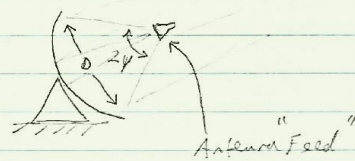
Aperture Antennas

For high gain and narrow beamwidth at low cost, use

- High Frequency (why?)
- Parabolic dish

With typical "illumination"
i.e. $2\psi = \text{BW}_{\text{feed}}$

$$\text{BW}_{\text{dish}} \approx 70^\circ \frac{\lambda}{D}$$



Example: 2m dish @ 10 GHz

$$\Rightarrow \text{BW} = 70^\circ \frac{30\text{cm}}{2} = 1.1^\circ$$

Recall that narrow beam concentrates power
resulting in high Effective Isotropic Radiated Power

$$\text{EIRP} = P_t G_t$$

← Directivity gain of dish
← Transmitter output power

Can be shown that

$$G_t \approx 0.5 \left(\pi \frac{D}{\lambda} \right)^2$$

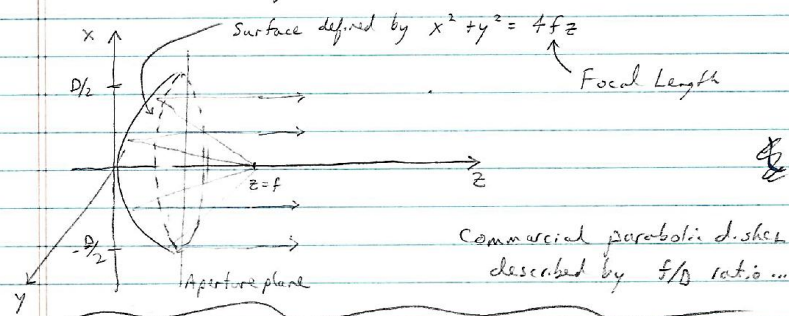
Example: 2m dish, 10 GHz, $P_t = 1\text{W}$

$$\Rightarrow G_t = 0.5 \left(\pi \frac{2}{0.03\text{m}} \right)^2 = 44,000$$

$$\Rightarrow \text{EIRP} = 44\text{ kW}!$$

Safety concerns! ?

Dish Geometry

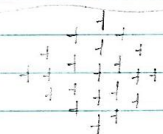


If dish "feed" antenna is at focal point ($z=f$),

- All rays leave dish parallel to z-axis
- All path lengths are equal $\Rightarrow$ equal phase

$\Rightarrow$ Field is uniform throughout aperture (if uniformly illuminated)

$\Rightarrow$ Similar to field from (2D) discrete array



$\Rightarrow$ Similar for field pattern

1-D Dipole array $\text{BW} = 50^\circ \frac{\lambda}{L}$

2-D Dish $\text{BW} = 70^\circ \frac{\lambda}{D}$ in both dimensions!

Non-uniform Excitation in aperture plane

Direction of plane of length

Feed Antenna Choices

Requirements: Fairly broad pattern, and good Front/Back ratio

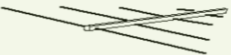
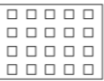
Antenna Examples

Antennas come in a nearly infinite number of shapes and sizes. Here are a few of the more common ones. Note that many commercial designs are variations or combinations of these.

"Non-directional" Types

Antenna	Impedance	Features
1) Halfwave Dipole	$73 + j0$ Ohms	Relatively isotropic, simple construction
2) Folded (halfwave) Dipole	$300 + j0$ Ohms	Similar to dipole. Higher impedance. Sometimes used in simple vertical arrays.
3) Quarterwave Monopole	$36 + j0$ Ohms	Similar to dipole. Ground-plane often abbreviated (e.g. case of cell-phone is ground plane)
4) Short Monopole	$R - jX$ $R \ll 36, X$ large	Physically shorter than quarterwave monopole, but requires resonating coil and/or matching network.
5) Loaded monopole	$R + j0$	Similar to short monopole, but resistive input impedance. Can be engineered to be nominally 50 Ohms.
6) Simple longwire	Varies Widely	Simple useful design at low freq (e.g. < 30 MHz). Often used for shortwave receivers.
7) Simple large loop or smaller resonant loop	Varies Widely	Simple, low-cost. Popular for indoor UHF TV antennas in "the old days"... Resonant loop used in KeyFobs today.
8) Small ferrite-core loop	$R + jX$	Physically small with large effective aperture. Used in portable AM broadcast receivers and other LF to HF products.
9) Microstrip patch	50 Ohms	Simple, low-cost. Useful mainly at high frequency (e.g. good for GPS).
10) Other	Inverted F, small loaded patches, etc.	Simple, low-cost designs for PC boards. May use high-K dielectrics to make antenna much smaller than a wavelength.

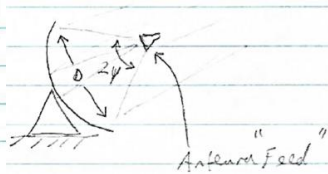
Directional Designs

1) Yagi-Uda		Moderate gain (10 dB) Good front-to-back ratio. Relatively simple construction.
2) Log-Periodic		Similar to Yagi-Uda, but broadband. Lower gain and less directivity.
3) Corner Reflector		Good "sector-coverage" (i.e. beamwidth of 90 to 120 degrees with excellent front/back ratio). Often used in cell-towers.
4) Horn		Gains to about 12 dB. Good illumination pattern for dish antennas.
5) Helix		Circular polarization. Moderate gain, and good illumination for dish antenna.
6) Linear, vertical array (tower-mounted dipoles)		Concentrates power toward horizon for max range. Used in public-safety (police/fire/etc.). Used in cell-towers when array embedded in corner reflector.
7) Parabolic dish		High-gain, narrow-beamwidth. Simple, low-cost construction.
8) 2-D Phased-array		High-gain, narrow-beamwidth, rapid-steering. Relatively high cost.

Feed Antenna Lectures and Labs

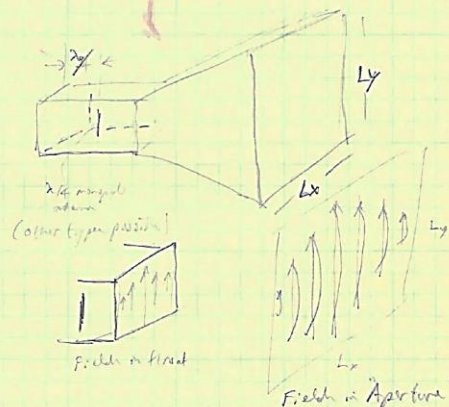
Antenna Feed Examples

- Patch
- Horn
- Helix
- Yagi-Uda
- Log Periodic
- Corner Reflector

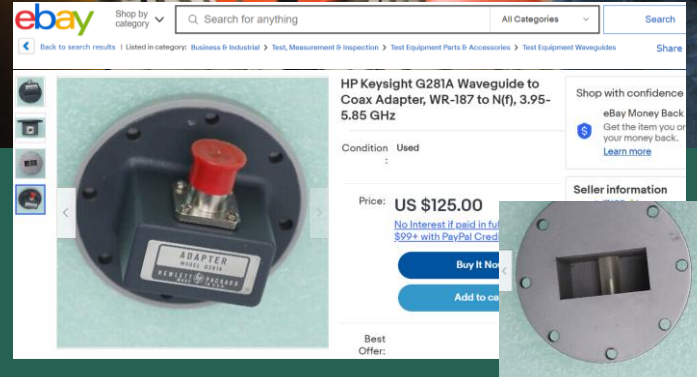
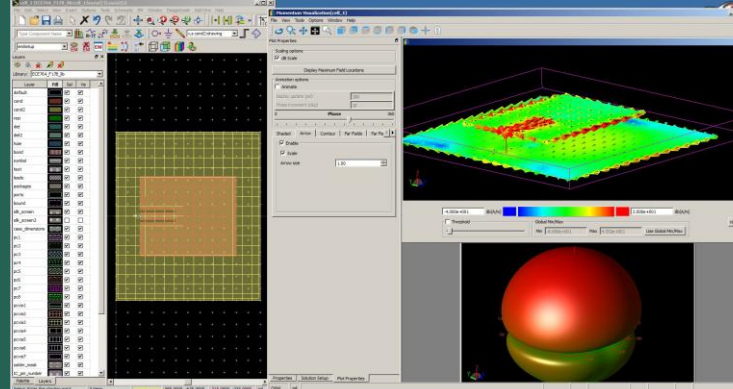


Antenna Feeder

Rect Horn Antenna

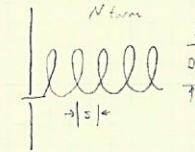


- Notes:
- $BW_{dB} = 2(50) \frac{L_z}{L_x}$ in B-plane (Azimuth Here)
 - $50 \frac{L_z}{L_y}$ in E-plane (Elevation Here)
 - Design Limited by "field curvature" in aperture
 - ⇒ Upper limit to L_x, L_y
 - ⇒ $BW \geq 15^\circ$
 - Circular geometry similar



Other useful directed antennas ("end fire type")

Helix



Design

$$\frac{3}{4} < \frac{TD}{\lambda} < \frac{4}{3}$$

$$\frac{5}{\lambda} = 0.25$$

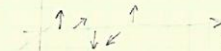
$$BW_{dB} = \frac{100^\circ}{\frac{\pi D}{\lambda} N}$$

$$G_0 = 4N \left(\frac{\pi D}{\lambda} \right)^2$$

$$R_{app} = 140 \frac{TD}{\lambda} \Omega$$

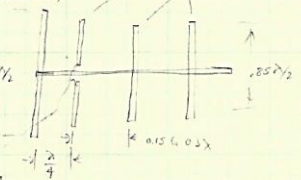
need mat

Note: • Fields are circularly polarized (Rotation is propogation)



- 2 Types - LH circular & RH circular. Must get it right!
- Need matching network

Yagi-Uda



Corner Reflector



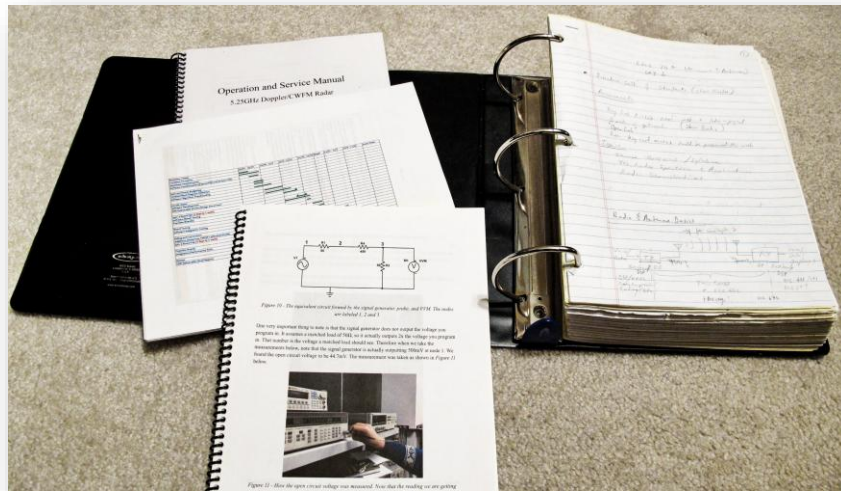
See other design books for more info.

- $BW_{dB} = 50$ to 20°
- $G_0 = 8$ to 13 dB
- $R_{app} \approx 10$ to 60Ω
- Also made in vertical Arrays for cell tower

Today's "Lecture" Topics

Announcements

Review material from prior Episodes



Topics

✓ ***Phased Arrays (Theory and Measurement)***

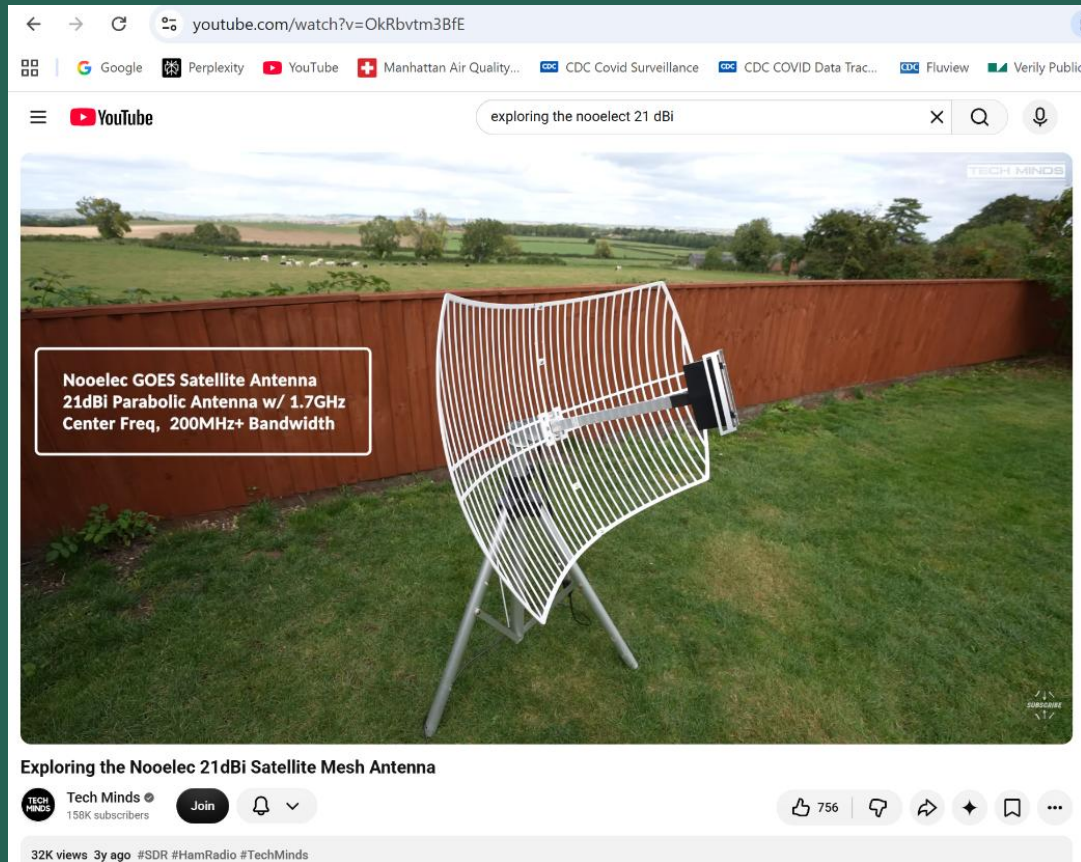
✓ ***Benefits of using Microwaves***

Derivation of Patterns and Beamwidths

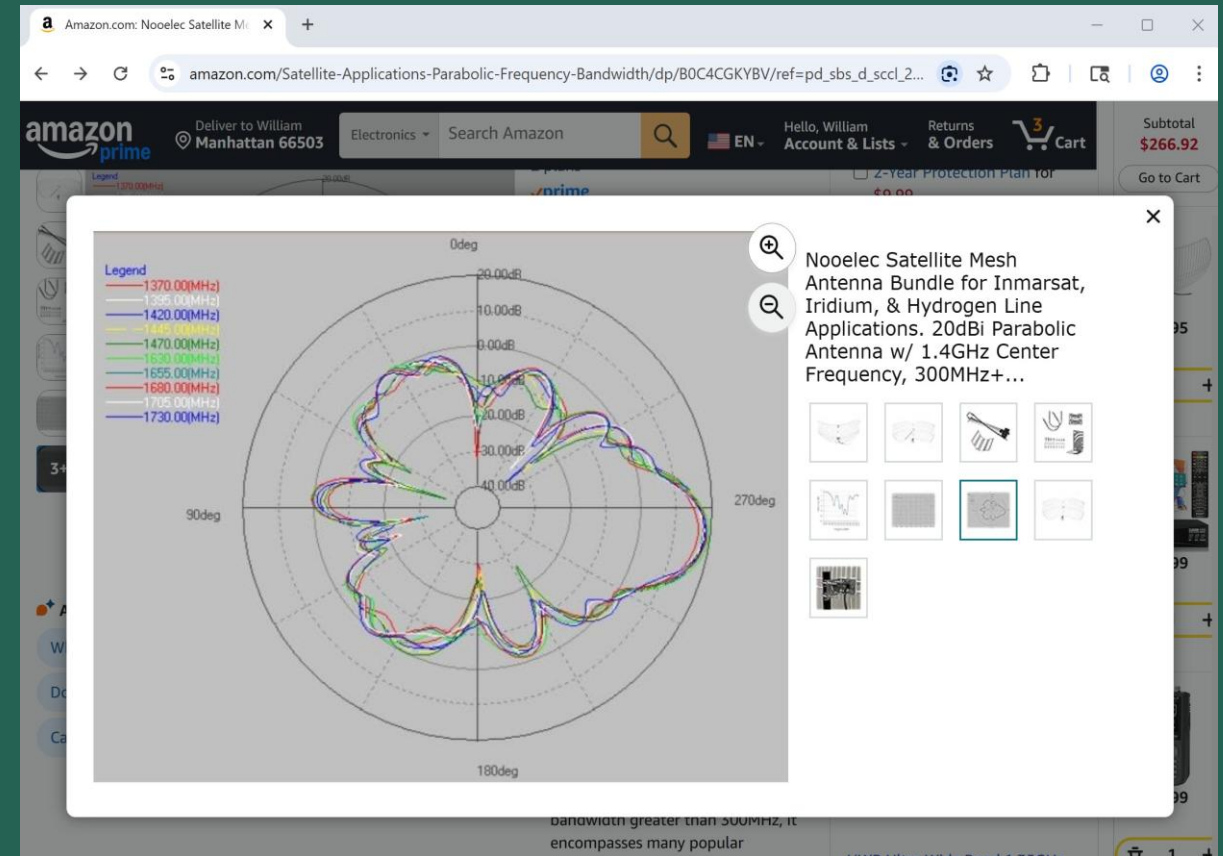
- ***1D Discrete Arrays and Fourier Transform Effects***
- ***Extension to 2D Aperture Antennas***
- ***Dish antennas (Illumination-angle, gain, sidelobes, ...)***
- ***Feeds (Horns, Helix, Dipole/corner-reflector, ...)***

➡ ***Application to Real-World Products/Design***

Inexpensive Dishes with 1.7 or 1.4 GHz Feeds

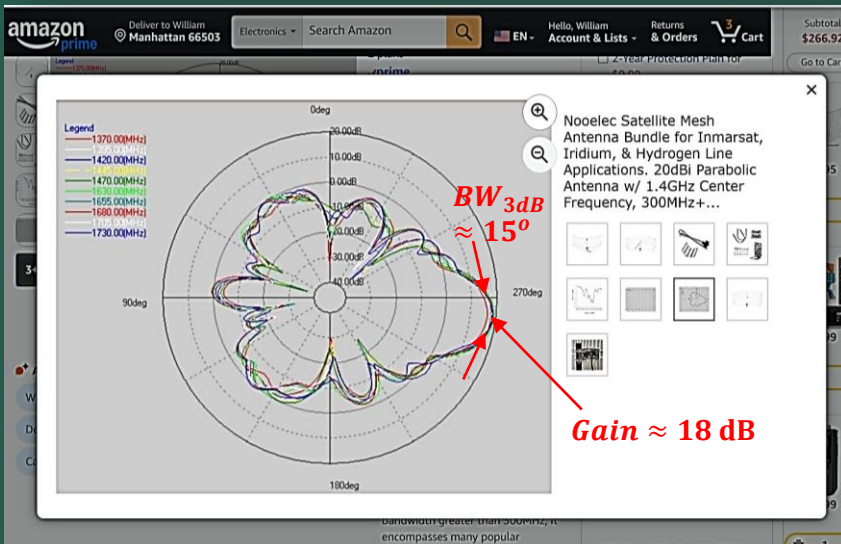
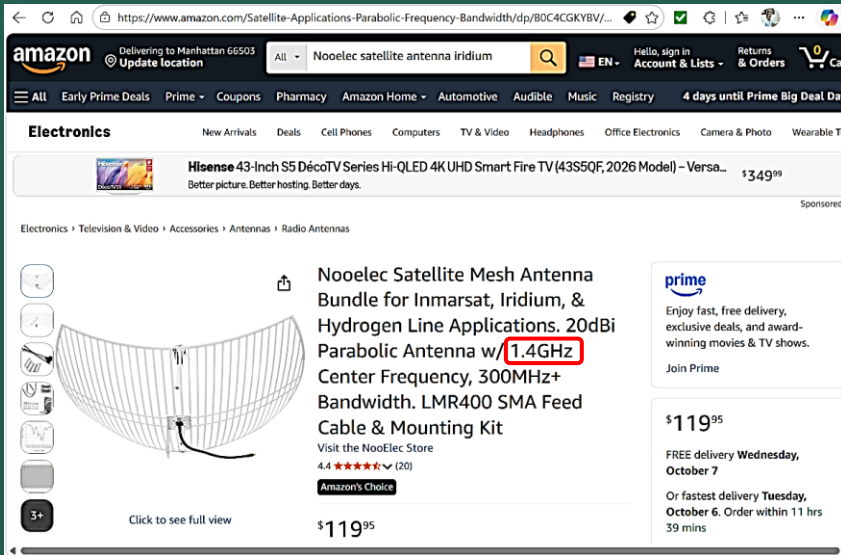


<https://www.youtube.com/watch?v=OkRbvtm3BfE>



<https://www.amazon.com/Satellite-Applications-Parabolic-Frequency-Bandwidth/dp/B0C4CGKYBV>

Theory vs Stated Performance



Stated: $f = 1.4 \text{ GHz}$

$\text{Size} \approx 0.99 \times 0.61 \text{ m}$ $BW_{3dB} \approx 15^\circ$ $G \approx 18 \text{ dB}$

Calculated: $\lambda = \frac{c}{f} = \frac{3E8}{1.4E9} = 21.4 \text{ cm}$

$$\theta_{az} \approx 70 \frac{0.214}{0.99} = 15^\circ \quad \theta_{el} \approx 70 \frac{0.214}{0.61} = 25^\circ$$

$$G \approx \frac{4}{\pi} (180^\circ)^2 \frac{1}{\theta_{el} \theta_{az}} = 110 \quad \rightarrow G_{dB} = 10 \log 110 = 20.4 \text{ dB}$$

Or:

$$G = \frac{4\pi A_e}{\lambda^2} \approx \frac{4\pi}{0.214^2} [(0.7)0.99][(0.7)0.61] = 81 \quad \rightarrow G_{dB} = 19.1 \text{ dB}$$

↑ ↑
Guestimated illumination factors

Multi-band Dish with LPDA Feed

Bolton "Long Ranger" Parabolic Antenna

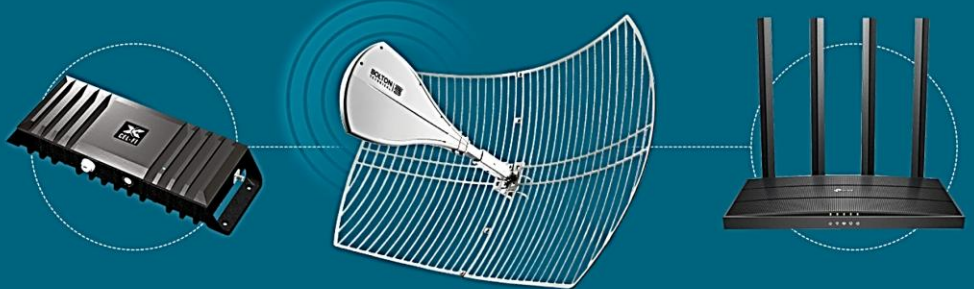
Amazon.com Shopping Cart x Amazon.com Shopping Cart x Amazon.com: Bolton Technical Lon... x +

amazon.com/gp/product/B08FBN5LJ7/ref=ox_sc_act_image_1?smid=A3H9JF6OQF5L9G&psc=1

↑ Top About this item Similar Product information Reviews

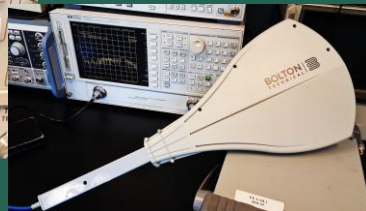
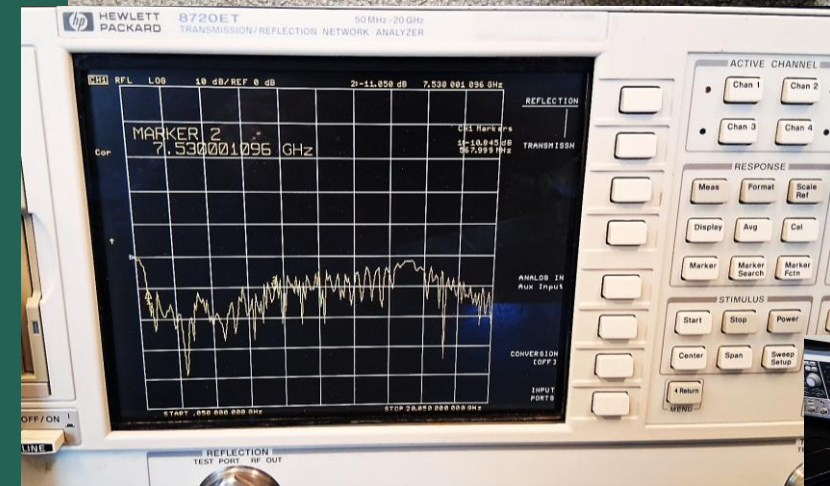
Bolton Technical Long Ranger Antenna (BT974822) |... \$199⁹⁹ Add to Cart

It works great with Cellular Signal Boosters and cellular Wifi Routers
**Cellular Signal Boosters and cellular Wifi Routers Sold Separately*



ELECTRICAL SPEC Spec Sheet Electrical Spec Mechanical Spec Radiation Pattern

Item No.	BT974822			
Frequency Range	MHz	600-960	1700-2200	3000-6500
Gain	dBi	8-15	20-28	10-26
Polarization	Vertical or Horizontal			
Beam width	Horizontal	deg	23	10
	Vertical	deg	35	13
VSWR		≤2.5	≤2.0	≤2.5
Front-To-Back Ratio	dB	>11	20	>10
Input Power, maximum	watt	100		
Impedance	ohm	50		



Experiments in Modifying Dish for 10 GHz Satellite TV

Alternative/LNB-matched “Offset-Feed” Dish

youtube.com/watch?v=USvucnj51HA

YouTube

Search

Ku Band Geostationary Satellite
Aiming, Blind Scanning, and Locking

Using a GTMEDIA V8 Finder 2
and a Small, Modified, 6 GHz Dish

Installation

gall19

1.48 FWHG 0.7601

DISEqC1.0 Disable

DISEqC1.1 Disable

22K On

LNB Power Auto

Motor Setup None

S 0%

Scan 0%

Power 22K 13V/18V Lock Charge

Credit: Wikipedia.org

Using GTMEDIA V8 in Blind Scan mode with small dish antenna

MegawattKS
14.1K subscribers

Analytics Edit video

31 Share Promote Ask

1,181 views Mar 19, 2026

This is a follow-on to a video where we overviewed the menus in the GTMEDIA V8 Finder 2. In this follow-on video, we illustrate modifications to a Long Ranger dish antenna to make it work at 10 to 12 GHz and then take the setup outside to find Galaxy 16 (and/or 19) satellite(s) using the GTMEDIA V8. Several details are shown that may not apply to everyone, but hopefully they are interesting, and the use of Blind Scanning is highly recommended for all. Special thanks to the Claude AI for useful discussions during this project - especially for recommending the formula for estimating the focal distance for the LNB feed mounting position :-)

BUY 1, GET 1 AT 5% OFF (add 2 to cart) See all eligible items and terms

39" ELECTORICA

39 Inch KU SATELLITE DISH ANTENNA For FTA LINEAR Inb 36 X 39 For 97 95 GALAXY 19

ELECTORICA (44842)
95.5% positive Seller's other items Message seller

US \$124.99
as low as \$42.64/mo with Klarna. Learn more

Condition: New

Quantity: 1 More than 10 available - 88 sold

Buy It Now

Add to cart

Add to Watchlist

Additional service available

☐ 2-year protection plan from Allstate - \$9.99

This one's trending. 88 have already sold.

People want this. 121 people are watching this.



LNB with circular
waveguide scalar ring



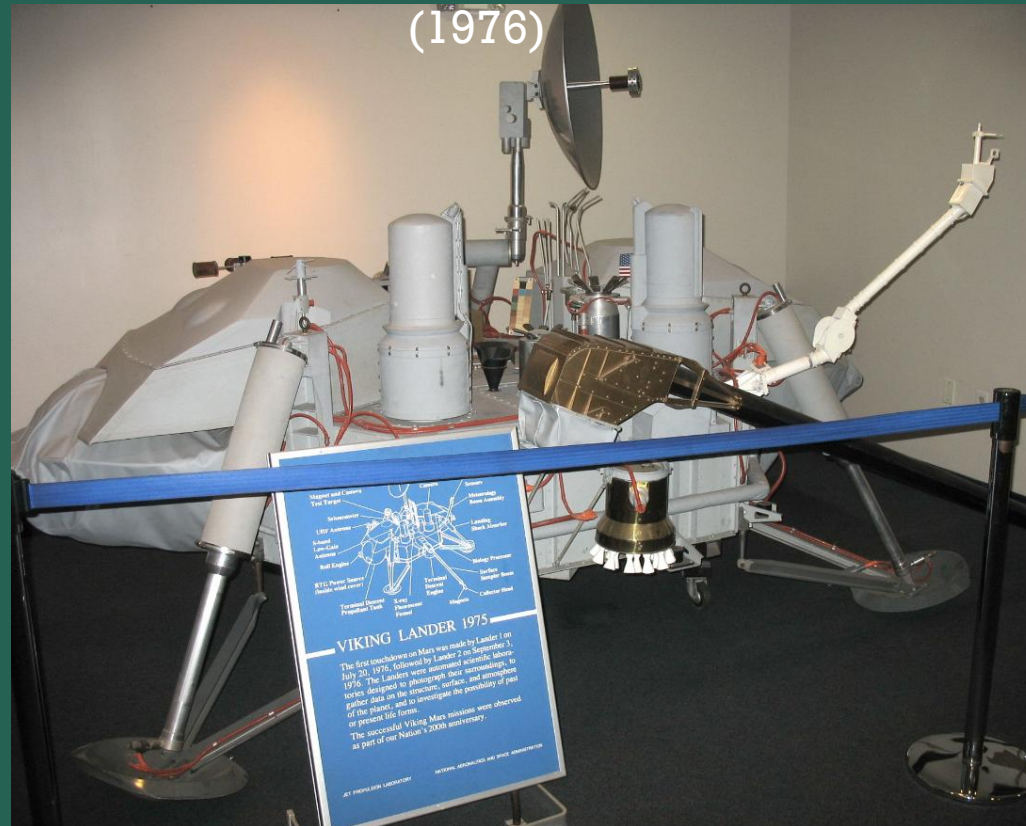
Early NASA Landers with Dishes

Kansas Cosmosphere
Museum in Hutchinson



Viking Mars Lander

(1976)



Lunar Lander and Rover

With dish using S-band helical feed

(1971)



Lunar Rover Relay Ops



CREW TRAINING MANUAL

LUNAR COMMUNICATIONS RELAY UNIT

LRTM-SY-1, Rev. D

30 March 1971

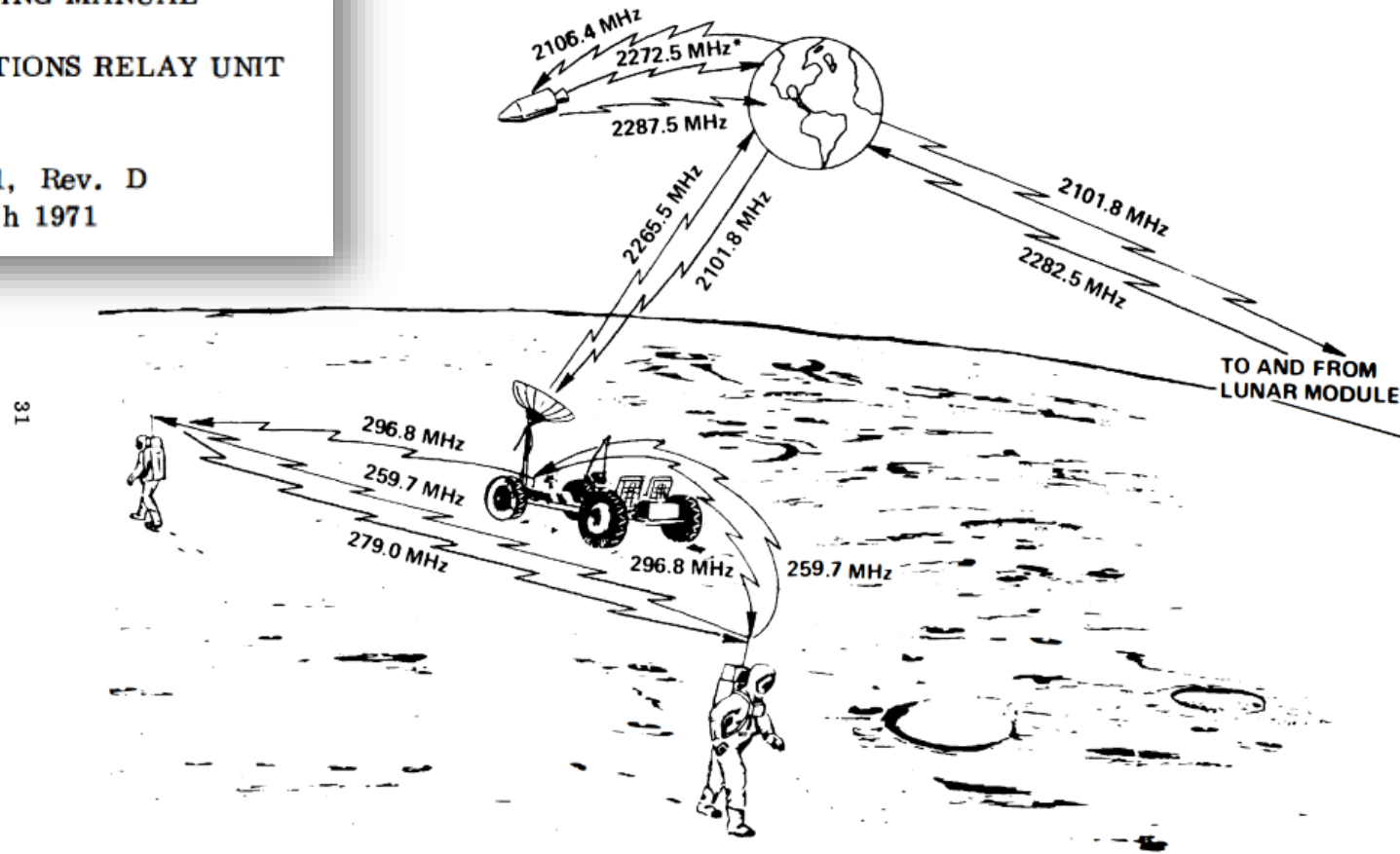


Figure 1-3. LCRU in Fixed Base Operation on LRV

Arrays of Dishes !

Celebrating MeerKAT's boundless scope for discovery
July 5, 2024

0:00

As South Africa's greatest astronomical project comes of age, researchers mark its achievements

The discovery of 49 new galaxies in a few hours, announced earlier this year, was the latest testament to the value of South Africa's MeerKAT telescope. Six years after its inauguration, it is finally proving to be much more than an iteration of the future giant radiotelescope Square Kilometre Array.



The 64-dish MeerKAT radio telescope array was built between 2014 and 2018 through South African government funding some system from the nearest town.

The recent trove of new galaxies — with dozens more waiting to be revealed in unpublished data — is a demonstration to the sensitivity it has reached. Institutions and researchers from around the world, including South Africa, Kenya, the UK, the Netherlands, Italy, the USA, France, Australia and Germany are lining up to use the instrument. A major technical update that started this year will further hone its sensitivity. And a whole generation of PhD students and postdocs trained in its operation have revitalized African astronomy.

MeerKAT consists of 64 antennas, each 13.5 metres in diameter, stretching over an area of eight kilometres. The project, 90 kilometres from Carnarvon in the Northern Cape, took a decade from conception to conclusion and was funded by the South African government.

There are eight large Survey Projects (LSP) under MeerKAT and 300 smaller projects. Since the publication of the MeerKAT team's first paper in 2019 in *Nature*, describing giant radio bubbles surrounding the Galactic Centre of the Milky Way, the affiliated researchers have produced more than 300 papers. Among the most recent is a study in

MEDIA RELEASES



A black hole in the Milky Way aims its jet at us

Share



A decade of impact: SARAO celebrates 10 years of Robotics sponsorship with epic regional wins by Northern Cape teams

Share

<https://www.sarao.ac.za/news/celebrating-meerkats-boundless-scope-for-discovery/>

<https://ngvla.nrao.edu/>



ngvla
Next Generation Very Large Array

About Science Technology Community News Broader Impacts Events Documents

Multimedia

Conference • December 7th, 2026

ngVLA-Mex 2026 Meeting



Conference • November 10th, 2026

ngVLA International Science Conference 2026

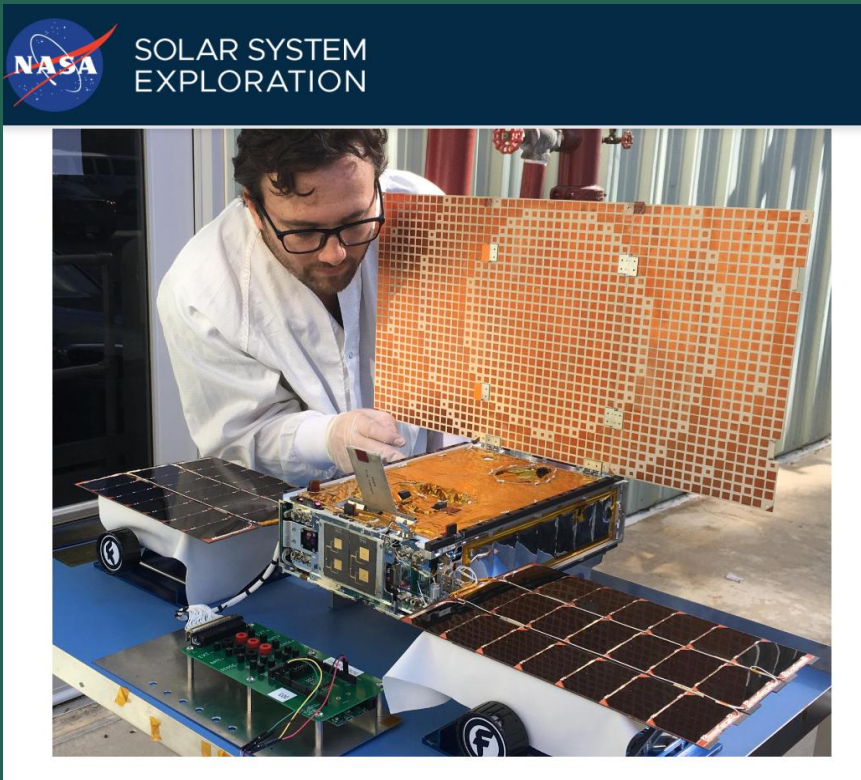


Workshop • October 5th, 2026

Third Workshop on German Science Opportunities for the ngVLA



Advanced 2-D Phased-Array Designs (Microwave Aperture Antennas)



<https://solarsystem.nasa.gov/missions/mars-cube-one/in-depth/>



<https://arstechnica.com/information-technology/2021/11/starlink-unveils-2nd-generation-satellite-dish-and-new-wi-fi-router/>



<https://www.youtube.com/watch?v=iOmdQnlnRo>

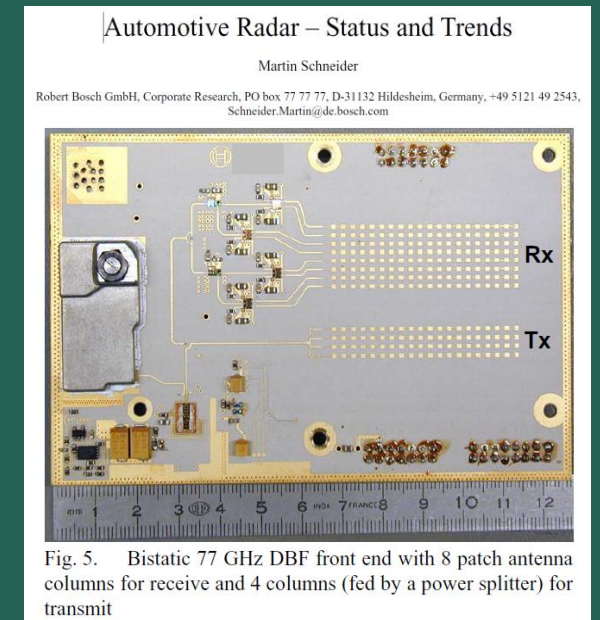


Fig. 5. Bistatic 77 GHz DBF front end with 8 patch antenna columns for receive and 4 columns (fed by a power splitter) for transmit

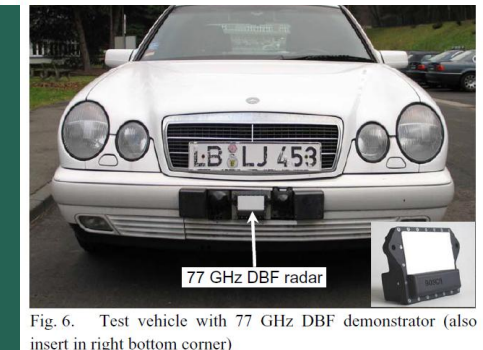


Fig. 6. Test vehicle with 77 GHz DBF demonstrator (also insert in right bottom corner)

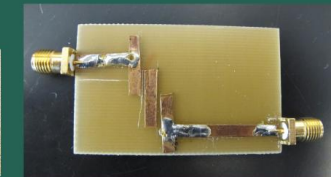
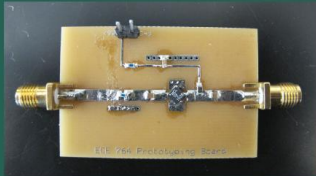
www.scribd.com/document/94178930/5-3-Radar-Sensors

Past and Future Episodes

Radio Design 201 #3 Microwave Circuits and Antennas Project-oriented Learning

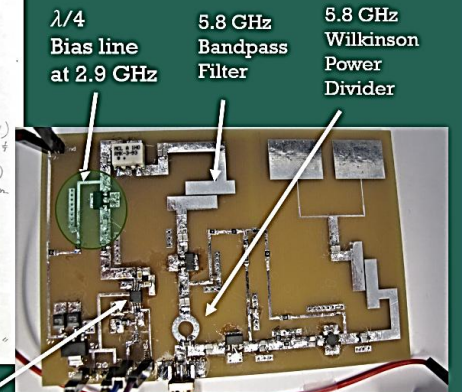
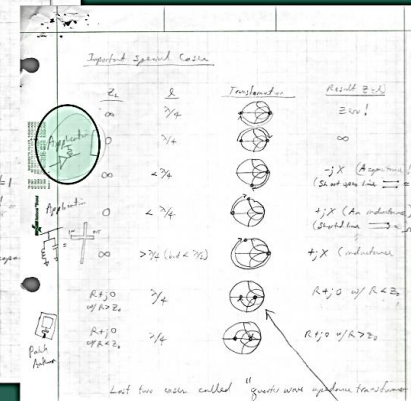
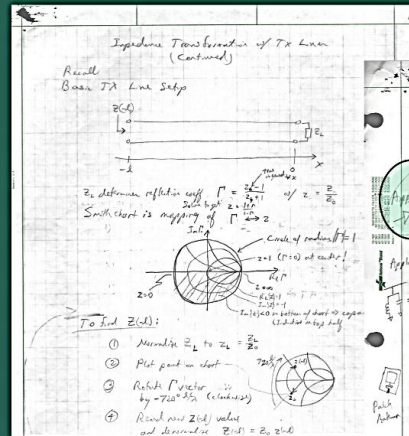


MegawattKS - YouTube



<https://ecefiles.org>

Microwave Circuits ... Where we're headed ...



2.9 GHz
VCO

Credit: KSU ECE Students

*Thanks For
Watching !*