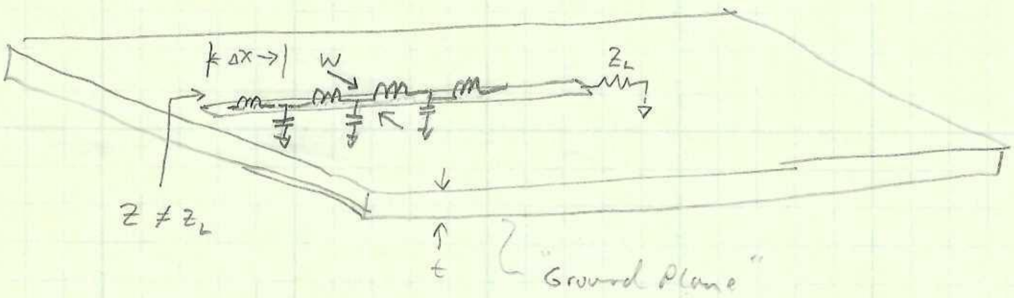
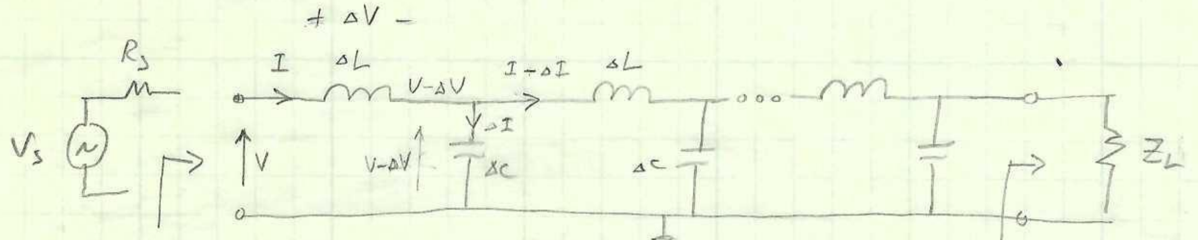
Intro to Transmission Lines

PC Board parasitics (one example of TX line effects)



Lumped Element model of PC board trace (A transmission line!)



$Z(-l) \triangleq \frac{V(-l)}{I(-l)} = ?$

$Z(0) = Z_L = \frac{V(0)}{I(0)}$

x = −l x = 0 x

complex phasor

we need $V(x)$ and $I(x)$ to get $Z(x) = \frac{V(x)}{I(x)}$

L/unit length → See EM Textbook

C/unit length →

$\Delta L = \mathcal{L}\,\Delta x$ $\Delta C = \mathcal{C}\,\Delta x$ $\mathcal{C} = \frac{\epsilon A}{d\,\Delta x} = \frac{\epsilon_0 \epsilon_r W \Delta x}{t\,\Delta x} = f\left(\frac{W}{t}\right)$

$\Delta V = -j\omega(\mathcal{L}\Delta x)\, I$ $\Delta I = -j\omega(\mathcal{C}\Delta x)\, V$

⇓ ⇓

$\frac{\partial V}{\partial x} = -j\omega \mathcal{L}\, I(x)$ $\frac{\partial I}{\partial x} = -j\omega \mathcal{C}\, V(x)$

$$\frac{\partial^2 V(x)}{\partial x^2} = -\omega^2 \mathcal{L}\mathcal{C}\, V(x)$$

2nd order P.D.E.

phasors
(complex) constants for fit boundary conditions amp/phase of voltage
phase variation w/ distance

Solve to find

V at location x is sum of 2 phasors & varies w/ x (since $e^{-j\beta x} \neq e^{j\beta x}$)

$$V(x) = V^+ e^{-j\beta x} + V^- e^{+j\beta x}$$

where $\beta = \omega\sqrt{LC} = \frac{2\pi}{\lambda}$ ← since $\Delta\theta$ must be 2π for $\Delta x = \lambda$

and $\lambda = v_p/f$ w/ $v_p = \frac{1}{\sqrt{LC}} = v_r\, c$ — velocity factor, speed of light

Solution also given:

$$I(x) = I^+ e^{-j\beta x} - I^- e^{j\beta x}$$

with $I^+ = \frac{V^+}{Z_0}$ $I^- = \frac{V^-}{Z_0}$

where $Z_0 = \sqrt{\frac{L}{C}}$ "characteristic impedance"

☆ **Interpretation:** V & I are sums of forward and backward traveling waves (FTW, BTW)
V^+, I^+ V^-, I^-

OPT

Why? $V(x,t) = |V^+| e^{j\theta^+} e^{-j\beta x} e^{j\omega t} + |V^-| e^{j\theta^-} e^{j\beta x} e^{j\omega t}$

$= |V^+| e^{j[(\omega t - \beta x) + \theta^+]} + |V^-| e^{j[(\omega t + \beta x) + \theta^-]}$

Time delayed $e^{j\omega t}$ ⇒ moving in +x direction

phase of sinusoid at x = 0

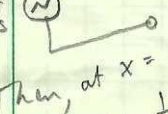
Time advanced ⇒ moving in −x direction

Finding V^+, V^- (Fitting Boundary Conditions)

$I(0)$, $V(0)$, Z_L, $x = -l$, $x = 0$

NOTE at load V, I in phase if $Z_L = R_L$. But not in phase at other locations ⇒ $Z \neq R_L$

Assume source is "matched" to Z_0:



Then, at $x = -l$
$V^+ = V_s \frac{1}{2}$

At $x = 0$, $Z = Z_L$

But $Z_L = Z\big|_{x=0} = \frac{V(0)}{I(0)} = \frac{V^+ e^{-j\beta 0} + V^- e^{j\beta 0}}{I^+ e^{-j\beta 0} - I^- e^{j\beta 0}}$

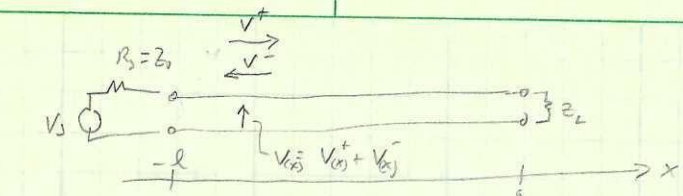
$$\Rightarrow Z(0) \equiv Z_L = \frac{V^+ + V^-}{\frac{V^+}{Z_0} - \frac{V^-}{Z_0}}$$

Solve to find:

$$V^- = V^+ \underbrace{\frac{Z_L - Z_0}{Z_L + Z_0}}$$

"Reflection coeff" Γ

Interpretation:



$$V^- = V^+ \Gamma_L \quad \text{where} \rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Backward Traveling wave (Dependent Variable)

Forward Traveling wave (Independent Variable) — "Because we can place a source at left side, to 'launch' V^+"

"Reflection coefficient"

NOTE: If $Z_L = Z_0$, $V^- \equiv 0$! (no reflection)

So, too

$$V(-l) = V^+ e^{-j\beta(-l)} \text{ \& } I(-l) = I^+ e^{-j\beta(-l)}$$

$$\Rightarrow Z(-l) = \frac{V(-l)}{I(-l)} = \frac{V^+}{I^+} = \frac{V^+}{V^+/Z_0} = Z_0 = Z_L \ !$$

Smith Charts

Recall $V^- = V^+ \Gamma$

or $\Gamma \triangleq \frac{V^-}{V^+}$

We found! $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

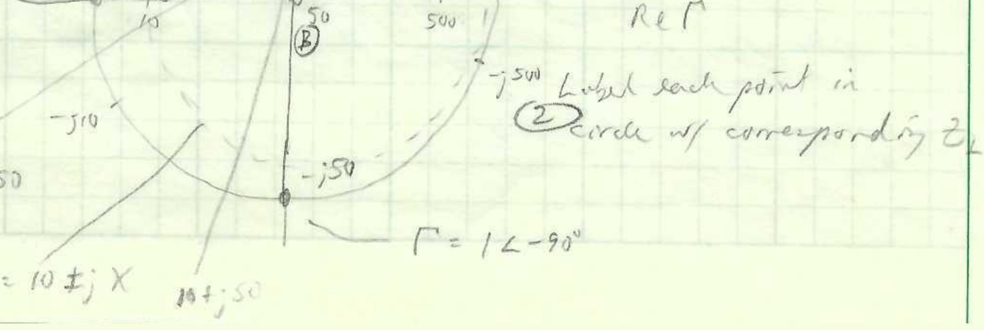
Can also get Z_L if Γ known (Γ easier to measure) $\quad Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$

☆ Smith chart is
- a graphical tool for mapping between Γ & Z
- a method for displaying impedance
- a method for finding $Z(l)$ given Z_L

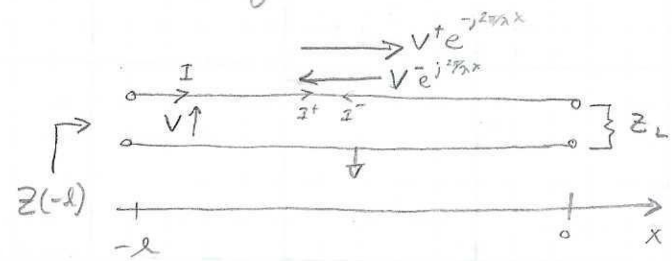
① On Polar Chart Find 0, 50, ∞, 10, 500, +j50, −j50, ±j10

10+j10, 10+j50, 10+j500, 10−j10 ...

② Label each point in circle w/ corresponding Z_L



Review of TX Line Equations:



$$V(x) = V^+ e^{-j\frac{2\pi}{\lambda}x} + V^- e^{j\frac{2\pi}{\lambda}x}$$

$$I(x) = I^+ e^{-j\frac{2\pi}{\lambda}x} - I^- e^{j\frac{2\pi}{\lambda}x} \quad \text{where } I^+ = \frac{V^+}{Z_0} \quad I^- = \frac{V^-}{Z_0}$$

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \text{typically } 50\,\Omega$$

$$\lambda = \frac{v_p c}{f} \qquad 0.5 \le v_p < 1 \qquad v_p = \frac{1}{\sqrt{LC}}$$

V^- found from V^+ at load by

$$\Gamma_L \triangleq \frac{V^-(0)}{V^+(0)} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

or, if Γ_L is known,

$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

☆ NOTE: Equation also apply at other points on line.

(Normalized) Smith Charts

What's a Smith Chart??
1) A graphical mapping between $\Gamma \leftrightarrow \bar{z}$
2) A format for displaying $\bar{z}$
3) A tool for finding $Z(-l)$ given Z_L
4) The coolest thing in the world
5) All of the above

Want to do Smith Chart for any Z_0

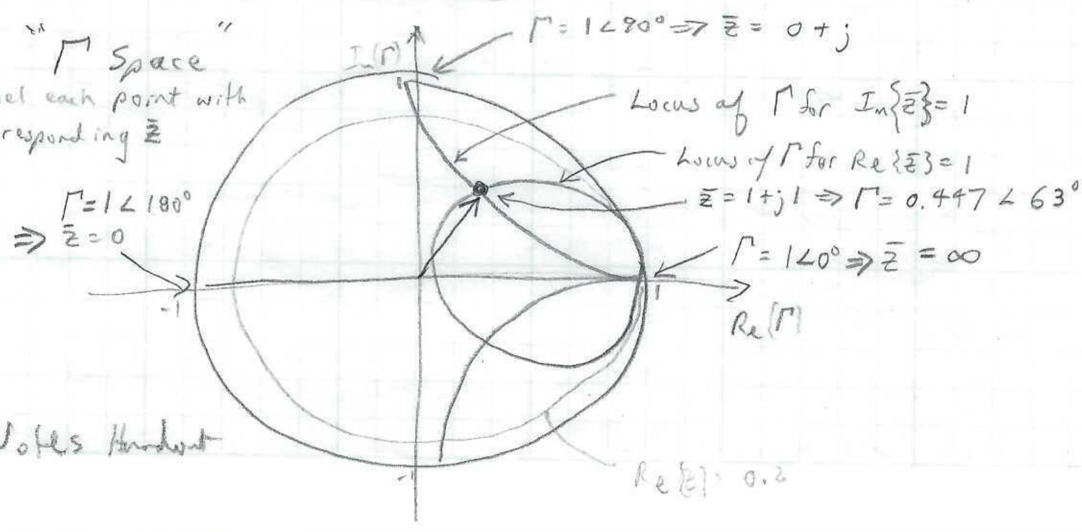
First normalize Z: $\boxed{\bar{z} \triangleq \frac{Z}{Z_0}}$

Then $$\Gamma = \frac{V^-}{V^+} = \frac{Z/Z_0 - Z_0/Z_0}{Z/Z_0 + Z/Z_0} = \boxed{\frac{\bar{z} - 1}{\bar{z} + 1} = \Gamma}$$

and $$\boxed{\bar{z} = \frac{1 + \Gamma}{1 - \Gamma}}$$

"Γ Space"

Label each point with corresponding $\bar{z}$



Show Davis Notes Handout

Finding z at $-l$ (distance of l "toward generator")

- Recall $V^+(x) = V_L^+ e^{-j\beta x}$, $V^-(x) = V_L^- e^{j\beta x}$, w/ $\beta = \frac{2\pi}{\lambda}$

Hence, $\Gamma(x) = \frac{V^-(x)}{V^+(x)} = \frac{V_L^-}{V_L^+} e^{+j2(2\pi \frac{x}{\lambda})}$

and $\Gamma(-l) = \Gamma_L e^{-j2(2\pi \frac{l}{\lambda})}$ — Phase change: $-360°$ for $l = \lambda/2$; $-180°$ for $l = \lambda/4$; -90 for $l = \lambda/8$

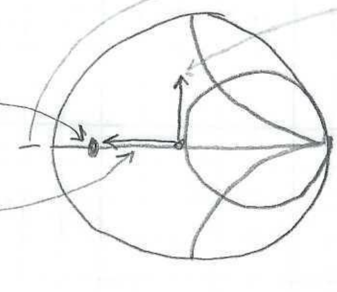
Example $l/\lambda = \frac{1}{8}$

On Smith Chart:

$Z_L = 10\,\Omega$ w/ $Z_0 = 50$

$\Rightarrow \bar{Z}_L = 0.2$

$\Rightarrow \Gamma_L = 0.667\angle 180°$

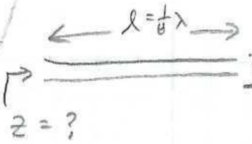


$\lambda = \frac{c\, v_r}{f}$

$= \frac{(3E8)(0.66)}{162E6} = 1.22\,m$

$\lambda/8 = 15\,cm$

At distance $l = \lambda/8$ toward generator

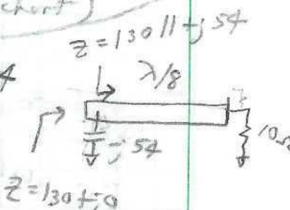


$\Gamma_{-l} = 0.667\angle(180° - 90°)$
$= 0.667\angle 90°$

$\Rightarrow \bar{Z} = 0.385 + j\,0.923$ (From eqn or chart)

$\Rightarrow Z = 19.2 + j\,46.2 \equiv 130.4 \,//\, +j54$

↑ change in R also — ↑ Series L



★ Show Amplifier

A BIG impedance transformation!

Important special cases

Z_L	l	$Z(-l)$	Comments
∞	$\lambda/4$ 	zero!	Open becomes a short!
0	$\lambda/4$	∞	Short becomes open
∞	$< \lambda/4$	$0 - jX$	Open becomes capacitor
∞	$\lambda/4 < l < \lambda/2$	$0 + jX$	inductor
0	$< \lambda/4$	$0 + jX$	Short becomes inductor
0	$\lambda/4 < l < \lambda/2$	$0 - jX$	capacitor
Z_0	Anything	Z_0	Unchanged since load is "matched" to line impedance

$V^-_{(x)} = \Gamma V^+_{(x)}$ $V_{tot} = V^+_{(x)} + V^-_{(x)}$ Incident Power $= \frac{|V^+|^2}{Z_0}$ why?

Measures of "Goodness of Match" to Z_0

Reflected Power $= \frac{|V^-|^2}{Z_0}$

SWR (or VSWR) (Voltage) Standing Wave Ratio

RL Return Loss

SWR (or VSWR)

$$SWR \triangleq \frac{\text{Max voltage on line}}{\text{min voltage on line}} = \frac{|V^+| + |V^-|}{|V^+| - |V^-|} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Good = Small e.g. SWR < "2:1" (i.e. $\Gamma < 0.333$)

Ideal = 1:1 why?

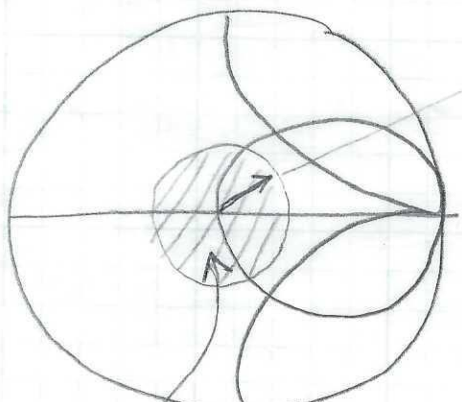
RL

$$\text{Return Loss} \triangleq 10\log\left[\frac{\text{Incident power}}{\text{Reflected Power}}\right] \text{ (in dB)} = 10\log\frac{\frac{|V^+|^2}{Z_0}}{\frac{|V^-|^2}{Z_0}}$$

Good = Big e.g. RL ≥ 12 dB

$\Rightarrow \Gamma \le 10^{\frac{-RL}{20}} \Rightarrow \Gamma \le 0.25)$

$RL = -20\log|\Gamma|$



$|\Gamma| = 0.333$ circle

RL ≥ 10dB, SWR ≤ 2 inside this region

(i.e. Z "close to" Z_0)

$|S_{11}|$ Like Return loss - but opposite sign in dB

$$|S_{11}| = +20\log|\Gamma|$$