Amplifier Power Gain

In radio frequency circuits, the power gain of an amplifier is often more important than the voltage gain. For example, LNAs designers must compare their amplifier's internal noise *power* to the power of the signal they amplify. PA designers, of course, are focused on their output power, power gain, and overall power efficiency This paper looks at calculation of power gain, and provides examples to illustrate important points.

Transducer Power Gain and Insertion Gain

The most commonly used definition of power gain is the so-called transducer gain G_T defined as:

$$G_T = \frac{P_{load}}{P_{avail}}$$

where P_{load} is the power delivered to the load by the amplifier, and P_{avail} is the power available from the source. In the situation where the source and load are both 50 Ohms, people often talk about this as the "Insertion Gain" of the amplifier.

 P_{avail} is the power delivered to the amplifier's input by the source, under the condition that the amplifier input impedance is matched to the source impedance (i.e. $R_i = R_s$ for the case of real impedances, or $Z_i = Z_s^*$ when working with complex impedances).

Assuming matched resistive impedances ($R_i = R_s$), the power available (power delivered to a matched amplifier input) can be written as:

$$P_{avail} = P_{in-matched} = \frac{V_i^2}{R_i}$$

The power delivered by the amplifier's output to a load resistance R_L (which may or may not be matched to the amplifier's output resistance R_o) is:

$$P_{load} = \frac{V_L^2}{R_L}$$

Combining these two expressions, we find:

$$G_T = \frac{V_L^2/R_L}{V_i^2/R_i} = \frac{V_L^2}{V_i^2} \frac{R_i}{R_L}$$

or

$$G_T = A_{V_{loaded}}^2 \frac{R_i}{R_L}$$

The maximum power will be delivered to the load by the amplifier if the load is matched to the amplifier's output impedance ($Z_L = Z_o^*$), or, for the case of purely resistive output and load impedances, $R_L = R_o$. Hence, the maximum transducer gain G_{Tmax} occurs when the output is matched as well as the input.

Examples:

Consider the case of a common-base amplifier with $g_m = 1/35$ ($R_i = 350$ hms), and $R_o = R_C = 1800$ Ohms. Assume that the input is matched to the source (i.e. $R_i = R_s$). The power gain for this amplifier under conditions of various load resistances is examined below.

Case 1: High impedance load

Suppose that the amplifier is hooked to a very high impedance load such as the input to an oscilloscope with $R_L = 10$ MOhm. Then:

$$A_{Vno-load} = g_m R_c = 1/35 \ 1800 = 51.43$$

and

$$A_{\text{Vloaded}} = A_{\text{Vno-load}} \frac{R_L}{R_L + R_o} = 51.42$$
 (34.2 dB)*

which is quite healthy. However, the power gain G_T is found to be only:

$$G_{\rm T} = A^2_{\rm Vloaded} \, \frac{R_i}{R_L} = 51.4^2 \, \frac{35}{10^7} = 0.01 \, (-20 \, {\rm dB})^*$$

Case 2: Low impedance load

Now suppose the amplifier output is connected directly to a piece of RF test equipment with a 50 Ohm input impedance. The loaded voltage gain is now:

$$A_{\text{Vloaded}} = A_{\text{Vno-load}} \ \frac{50}{50 + 1800} = 1.39 \tag{2.9 dB}$$

which is quite low. The power gain in this case is also low:

$$G_{\rm T} = V^2_{\rm Vloaded} \ \frac{R_i}{R_L} = 1.39^2 \ \frac{35}{50} = 1.35$$
 (1.3 dB)*

Case 3: Matched load

Finally, suppose that R_0 is matched to R_L (perhaps using an impedance matching network to transform a 50 Ohm load to 1800 Ohms). Then, the voltage and power gains are found to be:

$$A_{\text{Vloaded}} = A_{\text{Vno-load}} \frac{1800}{1800+1800} = A_{\text{Vno-load}} \frac{1}{2} = 25.7$$
 (28.2 dB)*

and

$$G_{\rm T} = G_{\rm Tmax} = A^2_{\rm Vloaded} \, \frac{R_i}{R_L} = 25.7^2 \, \frac{35}{1800} = 12.86 \, (11 \, \rm dB)^*$$

^{*} Recall that voltage gain in dB is defined as 20 log (A_{V}) and power gain in dB is 10 log (Gp).

The two are equal if and only if the input impedance equals the output impedance, which is not true here.