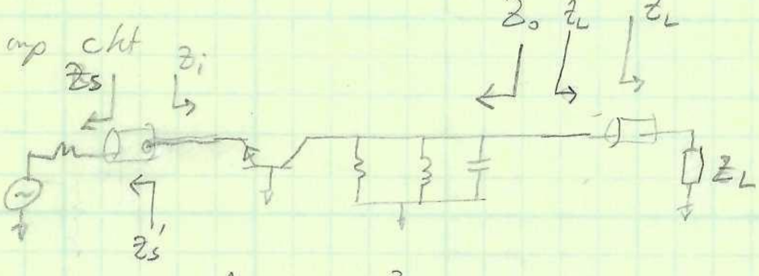
Show Power Gain Handout

9/19

Impedance matching ~~stuff~~

Consider RF amp ckt



$A_V = g_m Z_L$

$G_P = ?$

Problems: 1) Coax modifies Z_L (Varies w/ length of coax)

2) Need $Z_i = Z_S^*$ & $Z_L = Z_0^*$ for good power gain

Solns. 1) Use TX lines with proper termination

2) Add impedance matching networks (MNs)

$g_m R_L$

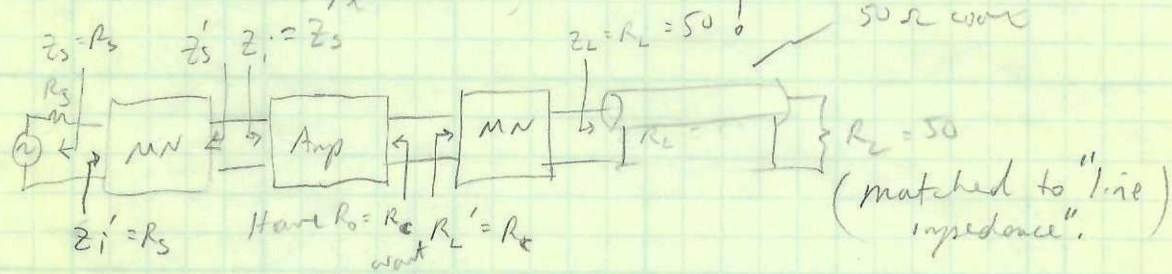
$\frac{V_L^2/R_L}{V_i^2/R_i}$

$A_V^2 \frac{R_i}{R_L}$

$(g_m R)^2 \frac{R_i}{R}$

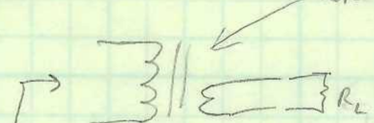
$g_m R$

Input Z must be 50Ω



Low freq Z matching network ($f \lesssim 100MHz$ at PCB level)

hard to get good cores above ~100MHz



$R_L' \approx \left(\frac{N2}{N1}\right)^2 R_L$ why?

Because $V_2 = \frac{N_2}{N_1} V_1$

And for power conservation,

$V_2 I_2 = V_1 I_1 \Rightarrow I_2 = \frac{V_1}{V_2} I_1 = \frac{N_1}{N_2} I_1$

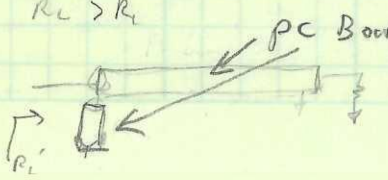
So $Z_2 = \frac{V_2}{I_2} = \frac{N_2/N_1 \, V_1}{N_1/N_2 \, I_1} = \left(\frac{N_2}{N_1}\right)^2 Z_1$

High freq matching networks

Lumped

$f \lesssim 2GHz$

ECE 662 :)



Distributed

$f \gtrsim 2GHz$

ECE 764

L matching Network Design

why OK? Because BP systems { 10.7 MHz ± 0.1 ; 100 MHz ± 10

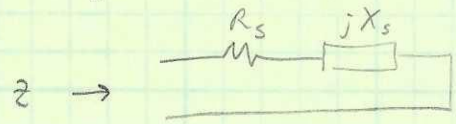
Background: Series/Parallel Equivalent Circuits

Consider an arbitrary Z

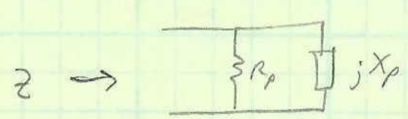
$$Z = R + jX$$

Theorem:

At a given freq Z can be "realized" either in series form



or in parallel form



Proof: "(Constructive)"

Series Form: (Easy) Set $R_s = R$, $X_s = X \Rightarrow L = \frac{X}{2\pi f}$ or $C = \frac{1}{2\pi f X}$

Parallel Form: (Harder)

$R+jX$

$Y = \frac{1}{R+jX}$

Work with Admittance value

$$Y = \frac{1}{Z} = \frac{1}{R+jX} = \frac{R-jX}{R^2+X^2} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$$

Compare with parallel circuit: $Y_{in} = \frac{1}{R_p} + \frac{1}{jX_p}$

To be same, we need:

$$R_p = \frac{R^2+X^2}{R} \quad \& \quad X_p = \frac{R^2+X^2}{X}$$

Series/Parallel Equivalents

Compare w/ series ckt

$$\boxed{R_p \equiv \frac{R_s^2+X_s^2}{R_s} \qquad X_p \equiv \frac{R_s^2+X_s^2}{X_s}}$$

Summary

ξ Alternative Formulation

Define $\boxed{q = \frac{X_s}{R_s}}$

Then $\boxed{R_p = (1+q^2)\, R_s \qquad X_p = \frac{1+q^2}{q^2}\, X_s}$

$\boxed{\text{Can also show: } \frac{R_p}{X_p} = q}$

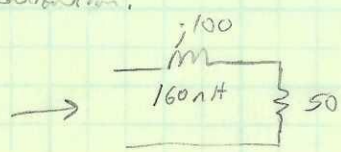
"Compare w/ previous formulas for Q_p, Q_s!"

Example:

$1+\frac{1}{q^2}$

Consider $Z = 50 + j\,100\ \Omega$ at 100 MHz

Series Realization:

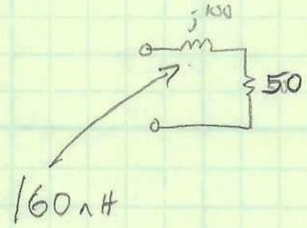


Parallel equivalent (at 100 MHz)

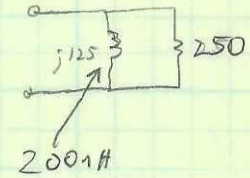
$$q = \frac{X_s}{R_s} = \frac{100}{50} = 2$$

$$R_p = (1+q^2)\, R_s = (5)(50) = 250\,\Omega$$

$$X_p = \frac{1+q^2}{q^2}\, X_s = \frac{5}{4}\, 100 = 125\,\Omega$$



$\equiv$



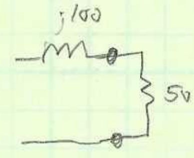
HANDOUT

Z matching with L networks

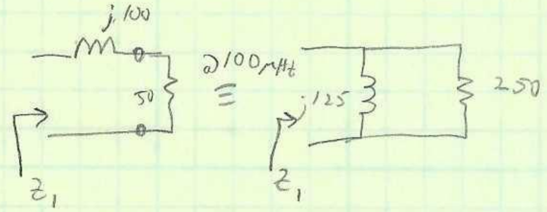
Example

Convert 50 Ω to 250 Ω at 100 MHz

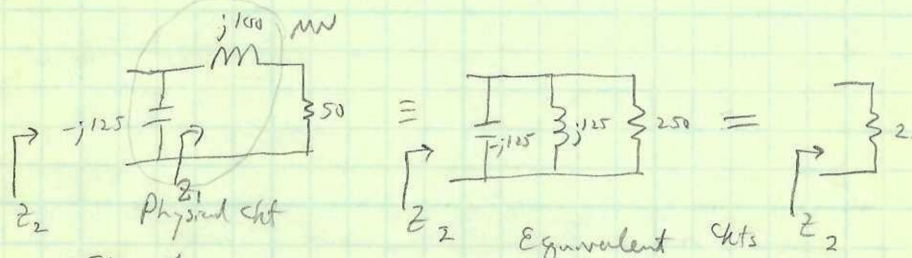
Step 1
Add series reactance



Step 2
View as parallel form (see prev. example)



Step 3
Resonate out parallel equivalent reactance



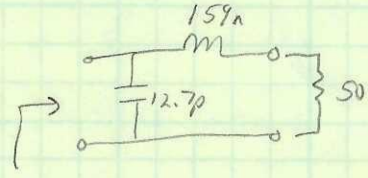
Step 4
Convert reactances to L, C values

$X_L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f}$

$= \frac{100}{2\pi \, 1E8} = 159\,nH$

$X_C = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_C}$

$= \frac{1}{2\pi (1E8)(125)} = 12.7\,pF$



$Z = 250\,\Omega + j0$

@ f_0 only

Step 5 check for practicality (PCB level)

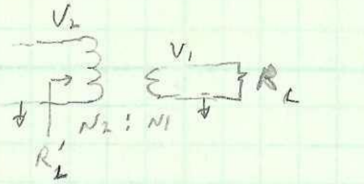
$C \gtrsim 1\,pF$ ✓

$L \gtrsim 10\,nH$ ✓

Additional MN Topics

why does this work?

Recall tformer Z transformation



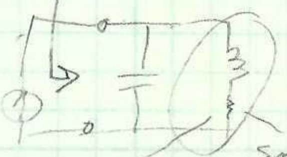
By power conservation, must have

$$\frac{V_2^2}{R_L'} = \frac{V_1^2}{R_L}$$

$$\Rightarrow \frac{R_L'}{R_L} = \left(\frac{V_2}{V_1}\right)^2 = N^2$$

$\equiv R_p \| -jX_C \| jX_L'$

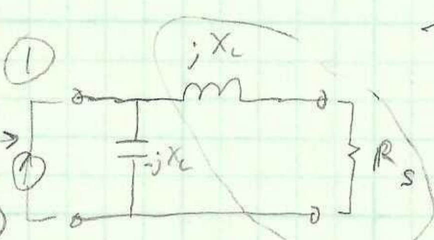
② $\equiv$



Small R $\Rightarrow$ low loss

Now look at resonant circuits

① If R_S small ($\ll X_L$), looks like (lossy) parallel tuned ckt w/ $Q = Q_p = \frac{R_p}{X_p}$



This side is lossy series RL circuit w/ $Q_S = \frac{X_S}{R_S}$

Small R for $Q \gg 1$

Recall from physics, $Q = \frac{2\pi E_{pk}}{E_{loss/cycle}}$

By power conservation $Q_S = Q_p = Q$

But $Q_p = \frac{R_p}{X_p}$ ← Large for $Q \gg 1$

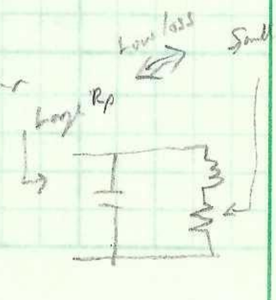
since $X_L' \approx X_L$ if $Q \gg 1$

Hence $Q \approx \frac{R_p}{Q R_S}$ ← X_L

$$\Rightarrow \frac{R_p}{R_S} \approx Q^2$$

NOTE: Z match network behaves like Tformer with $Q \approx \frac{N_2}{N_1}$

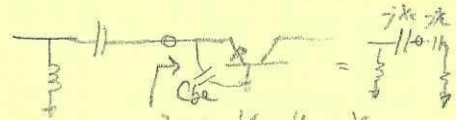
$\Rightarrow$ V higher on parallel side!



Additional MN Topics

- Matching to Complex Load

Matching to Complex loads:

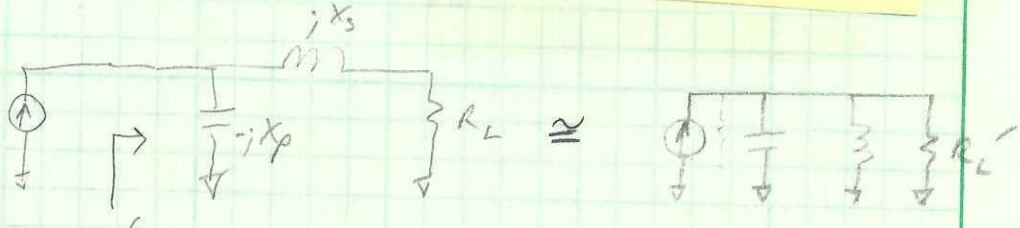


$Z_{in} = 1/y_{11} \parallel -jX_{cbe}$

convert to series form and absorb C into MN

- Bandwidth & "Loaded Q"

MN driven w/ I source

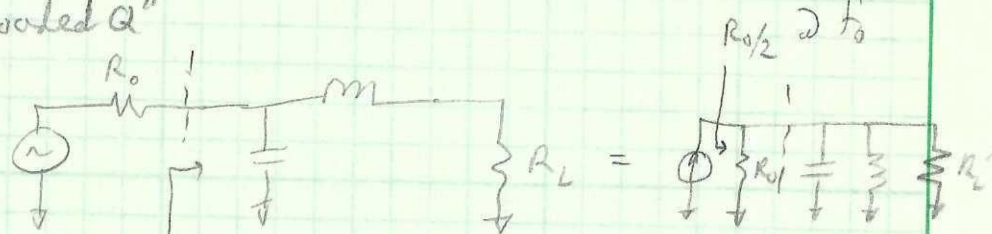


$R_L' = R_L(1+q^2)$

$BW = f_0/Q$ w/ $Q = \frac{R_L'}{X} \equiv q$

So $BW = f_0/q$ <u>in this case</u>

MN driven from matched source and definition of "Loaded Q"



$R_L' = (1+q^2)R_L$
$= R_0$ (for max G_p)

$BW = \frac{f_0}{Q_{Loaded}}$ w/ $Q_{Loaded} = \frac{R_0' \parallel R_0}{X_p}$

$= \frac{1}{2}$ Q unloaded

$= \frac{1}{2} q$

$\Rightarrow$ 2× bandwidth of unloaded case

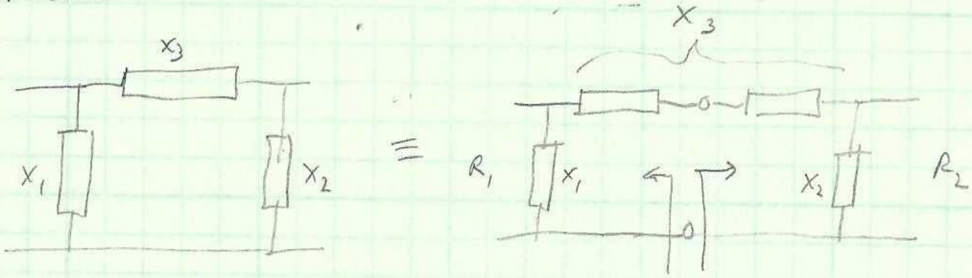
NOTE: Inductor has losses and will degrade Q & BW more

See handout on "Component Parasitics"

Additional MN Topics

Other MN types:

Pi Network



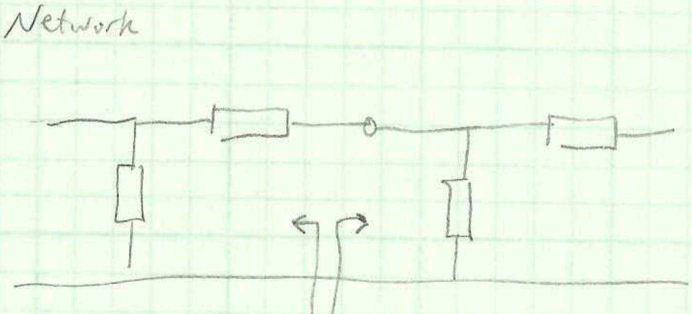
R_{int} = arbitrary

But must be < R_1 & R_2

T Network

Advantages –
- Often used for "tuners"
- may allow better L, C values
- narrower bandwidth realizable

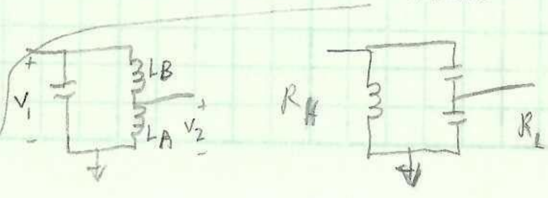
LL Network



$R_2 > R_{int} > R_1$

Advantages
- Broader bandwidth

<u>Others</u> Tapped L or C
- View as Pi Network, or
- Think of as V divider



- Z ratio = X ratio squared

$R_L = \left(\frac{X_A}{X_A + X_B}\right)^2 R_H$

iff $Q >> 1$